A Simple Model of Parental Influence

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Abstract. We study a model in which lifetime individual felicities are derived from both present and past consumption streams. Each of these streams is discounted, the former forward in the usual way, the latter backward. *Parental influence* refers to a state of affairs in which an individual at date t evaluates consumption programs according to some weighted average of his own felicity (as perceived at date t) and that of a "shadow parent" at some date $T > t$). This simple model can be used, among other things, to show that parental influence may have a positive impact on savings, that individuals may exhibit impatience across alternatives that are positioned in periods adjacent to the present, but patience across similar choices positioned in the more distant future, that such impatience is attenuated as an individual grows older, and that lifetime choice plans are generally time-inconsistent. The postulate of "backwards discounting" used in the model may also be of intrinsic interest.

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1 Introduction

In this paper, we suggest a theory of parental influence based on the following ingredients. Assume that lifetime individual felicities are derived from both present and past consumption streams. Each of these streams is discounted, the former forward in the usual way, the latter backward. *Parental influence* refers to a state of affairs in which an individual at date t evaluates consumption programs according to some weighted average of his own felicity (as perceived at date t) and that of a "shadow parent" at some date $T > t$). Think of the weight on the shadow parent as a measure of the degree of parental influence. We derive two implications of this model.

First, we argue that our theory of parental influence gives rise naturally to some of the experimental observations that motivate recent models of hyperbolic discounting (see, e.g., Ainslie [1991], Loewenstein and Prelec [1992], Loewenstein and Thaler [1989], Laibson [1997, 1998], and O'Donoghue and Rabin [1999]). The results are "natural" in the sense that our approach asks for no departures from the widely-used notion of geometric discounting. To be sure, wide usage does not mean that the geometric discounting hypothesis is the correct one. But it is a minimal hypothesis, one which has nice axiomatic features. In this sense it may be of interest to see how far one can go with the simplest model of discounting.

Despite geometric discounting, the model addresses the observational anomalies (thus far explained using hyperbolic discounting) because of a particular tradeoff. We postulate that an individual needs to balance his notions of happiness (as evaluated by his current self) with the notions that he knows he will value when he comes to some natural ending point — say, retirement. He knows that at that latter stage he will look *back* at his life. The consequent "reverse discounting" involved in looking back conflicts with the "forward discounting" that his current self uses.

Second, we use the model to argue that a society with greater parental influence will display a higher rate of savings. We tentatively suggest this result as one possible explanation for the phenomenon of higher savings rates in East Asia, where intergenerational ties are commonly felt to be stronger than in Western Europe and North America. At the very least, it should provoke some empirical questioning along these lines.

The theoretical argument for higher savings is made complicated, however, by the fact that future planned savings will generally not be faithfully executed when the future rolls around. This is the familiar time-inconsistency problem that accompanies hyperbolic discounting, and accompanies our model as well. The particular form it takes here is actually more acute than in the hyperbolic discounting model: individuals wish to consume high amounts today, and they pass on the burden of savings to the future even more disproportionately than in the hyperbolic discounting model. This is because effective future discount factors (across neighboring periods) are strongly influenced by the shadow parent, who discounts these periods backwards, as already discussed. This greatly expands planned savings rates in the future, but not so in the present.

This motivates a consideration of *equilibrium* savings, in the spirit of Strotz [1956], Phelps and Pollak [1968], and many others. By viewing future incarnations of the current self as different agents, one might view the savings problem as a game across incarnations. It turns out that in our model (with finite lifetimes and parametric restrictions on the one-period utility function), there is a unique subgame-perfect equilibrium, which generates (possibly time-dependent) savings rates at each date. We are able to track these savings rates as parental influence is exogenously varied. They unambiguously increase with the degree of such influence.

Our theory predicts that countries with closer family ties have higher savings rates and assign higher values to education (human capital investment). There is some anecdotal evidence in support of this phenomenon, though (especially given the importance of the subject) there has been very little work on it.[1]

[1]East Asia, where family ties are known to be relatively strong, is perhaps the region that would

Indeed, we are aware that we do not provide direct empirical support for our hypothesis. Nevertheless, what makes the theory compelling is that a simple, plausible postulate of parental influence *simultaneously* generates a number of different findings, ranging from an explanation of "discount-switching" in experimental psychology to a prediction of higher savings rates under certain cultural scenarios. Moreover, there are predictions (such as those in part (ii) of Proposition 1 below) that may be hard to generate from any *other* model.

Finally, we believe that the postulate of "backwards discounting" used in the model may be of independent interest. Suppose you are hired to manage an individual's consumption stream over a span of years, after which she writes you a letter of recommendation. Would you choose the consumption stream to maximize her (contractual) lifetime utility at the *initial* date, or at the final date? The two streams are different, if an individual at the end of the contract evaluates consumption profiles by discounting backwards in time. This simple observation may throw some (guarded) light on why parents disagree with children, on why politicians might inflate an economy at the end of an election cycle, and may even serve as an evolutionary explanation for why we might behave more patiently than our experimentally-deduced current discount factors suggest.[2] We complete the paper with some discussion of these issues.

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provide strong support for this point of view. For instance, despite low interest rates, Japan's savings rate is much higher than the rest of the industrialized countries. According to International Monetary Fund [1999a], in the last two decades, the interest rates (central bank discount rates) for Japan range from 0.5 to 6%, while the rates for the US range from 4.5 to 8.5%, and for the Europe area from around 3 to around 11%. For the same period, the gross national savings rates for Japan range from 30 to 35%, but range from 15 to 20% for the US, and 20 to 24 % in the Europe area. (See International Monetary Fund [1999b].) Hayashi [1997] tests various hypotheses explaining this significant difference in the savings rates using data on the US and Japan. One hypothesis that he mentioned but did not test is that the Japanese are more patient than the Americans. Our paper offers an explanation for why this may be true.

[2]After a first draft of this paper was written, we came across Caplin and Leahy [1999], which suggests

2 Parental Influence

It is well known that a parent's influence on the behavior of their children is very significant (see, e.g., Hess and Torney [1967], Bandera [1977] and Moschis [1987], among many others). Several aspects of this influence have been studied. Among other things, parents have a significant impact on such outcomes as their children's choice of career (Dryler [1998]), their focus on academic excellence (Salili [1994]), their perception of leadership (Harris and Hartman [1992]), their political attitudes (Hess and Torney [1967]), and on characteristics such as home ownership (Henretta [1984]). A recent study by Bregman and Killen [1999] show that parental influence was judged by adolescents and young adults to be most important when their decisions focused on short-term goals. Some recent literature in economics (Becker and Mulligan [1997], Bisin and Verdier [1998, 2000]) incorporates parental influence and upbringing as fundamental features of intergenerational economic interaction.

A particularly important target for parental influence — and this is what we emphasize here — is the attitude to consumption and savings. Moschis [1987, p. 77] summarizes the literature thus: "[T]here appears to be reasonably good supportive evidence that the family is instrumental in teaching young people basic rational aspects of consumption. It influences the development of rational consumption orientations related to a hierarchy of economic decisions delineated by previous writers...: spending and saving, expenditure allocation, and product decisions, including some evaluative criteria."

However, in our opinion, this form of influence extends far beyond *deliberate* attempts by parents to inculcate rudimentary notions of financial budgeting in their children, through the use of an allowance for instance. The appropriate channel of influence may be more akin to what Hess and Torney [1967] have termed *anticipatory socializa-*

that backwards discounting may have normative significance for the evaluation of consumption streams by a social planner, an observation we thoroughly agree with.

tion: the acquisition of attitudes and values about adult roles that have only limited relevance for the child but serves as a basis for subsequent adult behavior.[3] In an interesting and provocative essay, Brim [1966] views the socialization of an individual as a series of complex interpersonal relationships embedded in that individual. At the cost of some simplification, we might interpret this as stating that a particular personality is nothing more than the weighted combination of other personalities in the "cognitive neighborhood" of the individual in question. While staying away from the provocative analogies with interactive systems, we do notice that our model is a special case of this formulation, in which the attitudes of the "shadow parent" enter with some weight in determining current choices.

Turning, now, to the specifics of our formulation, we have chosen the simplest way in which a shadow parent situated at age 65, say, might disagree with the current self situated at age 30. We assume that all selves have the same preference *functional*, but that they discount both their future and their past.[4] This gives rise to a minimal but (in our opinion) cogent form of disagreement: the two selves will surely disagree on intertemporal decisions to be taken between the age of 30 and 50, for instance. Parental influence refers to a state of affairs in which the shadow parent's preferences have been partly internalized by the current self.

[3] As Ward [1974] describes it, such influence might be embodied in "implicit often unconscious learning for roles which will be assumed sometime in the future".

[4] Our assumption that these two discount factors are the same is purely for convenience and affects none of the substantial points of the paper.

3 A Model of Parental Influence

3.1 Preliminaries

We consider a discrete time model in which indices such as t, s and T are used to designate integer periods. Let $\{c_s\}$ be a consumption sequence over these dates, and let u be a one-period utility function defined on consumption at every date.

Our main postulate is that a person at date t discounts both the past and the future. We use a common discount factor $\delta < 1$ for both directions of discounting. This is not at all necessary for the results that follow, but it helps in reducing notational clutter.

Parental influence can be modeled in several ways, and the particular route we choose is by no means general. But it seems to us that the exploration of additional generality will not detract from the main points that we have to make in this paper. We will suppose that a person at date t places some weight on the utility experienced by his current self, as well as some weight on a "shadow parent" which we denote by his self at some (possibly) future date $T(t)$.

Depending on whether time horizons are finite or infinite, $T(t)$ may or may not be persistently sensitive to t. Or it may be that $T(t)$ is simply a fixed number that's independent of t — an age such as 65 that is viewed as a private retirement age. In the exposition that follows we shall assume that the lifespan is finite and fixed at N (where $N \geq T(t)$, to be sure). Nothing of substance will change if we consider infinite-horizon models, but it can be argued that in the current context, the finite lifetime interpretation is possibly more compelling. [With infinite lifetimes, what is the sense of placing some additional weight on a finite future?]

Thus consider a person currently alive at date t. How might he evaluate lifetime utility starting from some sequence of consumptions $\{c_s\}_{s=0}^{N}$? He attaches a weight of α

to his "current self" at date t, who derives felicity

$$\sum_{s=0}^{N} \delta^{|t-s|} u(c_s),$$

while a weight of $1-\alpha$ is placed on the felicity of the shadow parent, which is

$$\sum_{s=0}^{N} \delta^{|T(t)-s|} u(c_s).$$

Combining these two expressions, we see that our agent's utility function may be written in the form

$$\sum_{s=0}^{N} d(t,s) u(c_s), \tag{1}$$

where $d(t,s)$ can be thought of as the *effective discount factor* applied by a person at date t to consumption at date s, and is given by

$$d(t,s) = \alpha\delta^{|s-t|} + (1-\alpha)\delta^{|T(t)-s|} \tag{2}$$

3.2 Time Preference and Time Consistency

This simple model generates certain observed features in a natural way. First, it creates time-inconsistency in individual decisions. This is almost immediate from examining the sequence of effective discount factors given by (2). They are not geometric.

To understand this better, consider a person at date t, with a shadow parent of *fixed age* $T > t$. He is asked to evaluate an intertemporal choice between dates s and $s+1$ (where $t < s < T$). This person (situated at date t) acts *as if* he places a weight of $\alpha\delta^{s-t}$ on his "current self" at date s. However, when date s actually rolls around, the weight has changed to α, while the weight on the shadow parent T remains as before. Now, the current self evaluates the choice by discounting $s+1$ at a factor δ relative to s, but the shadow parent does exactly the opposite — he discounts *backwards*, as it were. It follows that once the weights change, the evaluation of this choice will change. This is at the root of the time-inconsistency.

Moreover, the time-inconsistency is of a particular kind. It reveals a preference for *current* consumption (once the decision time comes around). This phenomenon, which has been well-documented, has often been captured by postulating non-geometric preferences. The best-known of these is the hyperbolic class of discount factors (see Laibson [1997, 1998] and Harris and Laibson [1999]). The hyperbolic class, indeed, accounts for the following empirical regularity: discounting seems to be more active for time delays that are situated in the immediate future, whereas delays situated at a more distant date are viewed more neutrally. Thaler [1981] probably makes the point in the cleanest way: a person might prefer an apple today to two apples tomorrow, but if the same choice is presented for neighboring days 100 and 101, the expressed preference may well be reversed. A postulate of hyperbolic discounting nicely fits some of these features (see Ainslie [1991], Loewenstein and Prelec [1992], Laibson [1997], O'Donoghue and Rabin (1999)).

It is of interest that our model of parental influence generates this phenomenon (and related observations) without departing from the fundamental presumption that all discounting is geometric. It is only as a consequence of parental influence that two different discounting streams are mixed, generating a sequence of nongeometric effective discount factors.

As an example, suppose that $t = 30$ and $T(t) = 65$, with weights α and $1-\alpha$. Notice that preferences over the very near future (say years 30 and 31) are governed largely by the (exponential) discount factor of the *current* self — the comparison $1 : \delta$ matters far more than the comparison $\delta^{34} : \delta^{35}$. However, as the delay is pushed into the future — say ages 50 and 51 — the bias towards the present exerted by the current self is increasingly compensated for by the bias towards the future exerted by the shadow parent, and the two effects cancel, *from the vantage point of the thirty-year old.* Finally, it is possible that our thirty-year old current self may indeed exhibit *negative* discounting for ages close to 65, as the preferences of the shadow-parent now outweigh the increasingly fragile comparisons of his current self over these distant time periods.

The same sort of argument also shows how this anomaly tends to disappear with advancing age. A fifty-year old with a 65 year old shadow will assign greater weight to the shadow for the comparison 55–56 than the 30 year old assigned to the comparison 35–36. This makes the fifty-year old appear more patient.

Thus, viewed through the lens of the parental influence model, the "true" discount factor δ is to be thought of as a "low" number, reflecting some innate predilection towards high impatience. This impatience is diluted by the presence of a shadow parent, because the shadow parent discounts "backwards" and compensates (partially) for innate impatience.

Notice, however, that the parental influence model does yield predictions that are qualitatively *different* from hyperbolic discounting. For instance, the moderating influence of the shadow parent disappears once we consider choices made at adjacent periods very late in life (these are discounted "forwards" by all concerned parties). Thus impatience should rise again for choices offered (to younger individuals) at hypothetical *late* adjacent periods. We are unaware of empirical evidence that addresses this phenomenon.

To formalize these ideas, recall the effective discount factor $d(t,s)$ from (2). For each t and $s \geq t$,

$$d(t,s) \equiv \alpha\delta^{s-t} + (1-\alpha)\delta^{|T(t)-s|}.$$

Let

$$i(t,s) \equiv \frac{d(t,s)}{d(t,s+1)}$$

measure the degree of "one-period impatience" that person t feels regarding adjacent choices at date s. Then we may summarize our discussion so far in the following way.

PROPOSITION **1** [1] *For each t, $i(t,s)$ is a declining function of s for all $s \in \{t, t+1, \ldots, T(t)-1\}$, but then jumps to its maximum value of $1/\delta$ (and stays there) as s crosses $T(t)$.*

[2] *If $T(t) - t$ does not increase in t, $i(t,t)$ is a nonincreasing function of t, strictly decreasing whenever $T(t) - t$ decreases in t.*

The first part of the proposition states that from the vantage point of a person at date t, the relative impatience across adjacent dates in the future declines as the future is made more distant, being highest for choices between "today" and "tomorrow". This translates precisely into what Loewenstein and Prelec [1992] call the *common difference principle*: "if a person is indifferent between receiving $x > 0$ and $y > x$ at some later time, ... then he or she will strictly prefer the better outcome if both outcomes are postponed by a common amount [of time]".

To get a sense of the "end points", consider both $i(t,t)$ (impatience regarding current delays) and $i(t, T(t) - t)$ (impatience concerning future delays). It is easy to see that

$$i(t,t) > 1 \text{ provided that } \frac{\alpha}{1-\alpha} > \delta^{T-t-1}, \tag{3}$$

which will always be true if the current self and the shadow parent are sufficiently separated in time (or if $\alpha > 1/2$ and there is *some* separation between current self and the shadow parent). On the other hand,

$$i(t, T(t)) < 1 \text{ provided that } \frac{\alpha}{1-\alpha} < \frac{1}{\delta^{T-t-1}}, \tag{4}$$

which will also be true if there is sufficient separation between the current self and the shadow parent. Taken together, (3) and (4) suggest the following possibility: a person at date t may be impatient (in a perfectly standard way) over adjacent periods in the vicinity of the present, while he actually exhibits *negative* discounting for future periods!

The second part of the proposition states that the proclivity of a person to be impatient over adjacent current choices is attenuated as that person grows older. Specifically, this tendency towards moderation is observed as long as the shadow parent "grows" more slowly than the current self. In particular, if the age of the shadow parent is fixed, observed patience must always increase in age. We are not sure whether empirical evidence for this sort of phenomenon exists.

Proof of Proposition 1. Note that for $s \in \{t, t+1, \ldots, T(t)-1\}$,

$$i(t,s) = \frac{\alpha\delta^{s-t} + (1-\alpha)\delta^{T(t)-s}}{\alpha\delta^{s+1-t} + (1-\alpha)\delta^{T(t)-s-1}} = w(t,s)(1/\delta) + [1-w(t,s)]\delta, \tag{5}$$

where

$$w(t,s) \equiv \frac{\alpha\delta^{s+1-t}}{\alpha\delta^{s+1-t} + (1-\alpha)\delta^{T(t)-s-1}} = \frac{\alpha}{\alpha + (1-\alpha)\frac{\delta^{T(t)-s-1}}{\delta^{s+1-t}}}.$$

It is easy enough to see from this expression that $w(t,s)$ is declining in s. It follows from (5) that $i(t,s)$ is less than $1/\delta$, and in addition must be a declining function of s, for fixed t, as long as $s \in \{t, t+1, \ldots, T(t)-1\}$.

For $s \geq T(t)$, note that

$$i(t,s) = \frac{\alpha\delta^{s-t} + (1-\alpha)\delta^{s-T(t)}}{\alpha\delta^{s+1-t} + (1-\alpha)\delta^{s+1-T(t)}} = 1/\delta,$$

and this completes the proof of the first part of the proposition.

To establish the second part, notice that that

$$w(t,t) = \frac{\alpha}{\alpha + (1-\alpha)\delta^{T(t)-t-2}},$$

which proves that $w(t,t)$ is declining in t as long as the conditions of the proposition hold. Using this information in (5), we see that $i(t,t)$ must decline as well. ∎

3.3 Parental Influence and Savings

We have already remarked on the inherent time-inconsistency present in a model of parental influence. This complicates, to some extent, our evaluation of the effect of parental influence on the rate of savings. A person at date t, far removed from his shadow parent, may currently engage in high consumption, and then *plan* on saving higher and higher fractions of his income as he gets into middle age and approaches the age of his shadow parent. Indeed, such plans are perfectly consistent with his preferences at date t. The problem is, he may want to revise these decisions as he gets older.

Thus it is to be expected that while a calculation of optimal intertemporal consumptions and savings may show initial periods of high *planned* consumption and intermediate periods of high *planned* savings, these features may not manifest themselves in *observed* behavior once time-inconsistency is taken into account.

The purpose of this section is to examine these matters by looking at both planned behavior and time-consistent (equilibrium) behavior.

The analysis that follows can, in principle, be conducted for all constant-elasticity utility functions. That is, we can explicitly solve the model (both in its planned and equilibrium incarnations) in this case. But the special case of logarithmic utility yields particularly sharp predictions that are amenable to easy computation, and so we shall stick to this particular formulation.

Moreover, we keep the background model of asset accumulation as simple as possible. We suppose that a person lives for N periods and that his shadow parent's age T (where $T \leq N$) is independent of his current age. As already discussed, let $u(c) = \ln c$ be his per-period utility function and c_t be his consumption in period t. The per-period interest rate is given by r and the (forward and backward) discount factor is given by δ. Assume that both lending and borrowing can take place at the rate of interest r. Assets are A_0 to begin with, and the agent receives an an exogenous income stream $\{y_t\}$ over the dates $0, 1, \ldots, N$.

Let A_t denote assets in period t. Then, if c_t is consumed at that date,

$$A_{t+1} = (1+r)(A_t + y_t - c_t).$$

Consider an agent at date t, planning his decisions for some period $s \geq t$. He seeks to maximize the expression in (1). Noting that the decisions up to $s-1$ have already been made, this is tantamount to maximizing

$$\alpha \sum_{\tau=s}^{N} \delta^{\tau-t} \ln c_\tau + (1-\alpha) \sum_{\tau=s}^{N} \delta^{|\tau-T|} \ln c_\tau.$$

Notice that — as far as the current self is concerned — we are still discounting back to date t.

3.3.1 Planned Consumption and Savings

We begin by ignoring the time consistency problem and simply map out the profile of optimal consumption and savings, viewed from the vantage point of a person at (normalized) date 0. Even though it is isn't hard to solve this problem by exploiting the usual Euler equations, we will use an approach based on value functions. This has two advantages. First, it will give us optimal policy *functions* for each period (not just consumption and savings values), making it easier for us to compare changing attitudes across time periods. Second, this is the approach that will be necessitated when we solve the equilibrium problem (the one that takes the possibility of time consistency into account).

To this end, it will be useful to keep track of two different "value functions". The first, which we denote by V_t, simply records the utility experienced by the current self from date t onwards; we write this discounted back to date t. That is,

$$V_t(A) = \sum_{\tau=t}^{N} \delta^{\tau-t} \ln c_\tau,$$

where in the sequel, the c's will be optimal consumption decisions *planned* by the agent *at date* 0. The second value function, which we denote by W_t, tracks the utility experienced by the shadow parent (over the period t to N). This is *always* discounted to period T. It is given by

$$W_t(A_t) = \sum_{\tau=t}^{N} \delta^{|\tau-T|} \ln c_\tau.$$

Notice that at date N, both these functions are trivially logarithmic, and are of the form

$$V_N(A) = a_N \ln(A + y) + b_N, \tag{6}$$

and

$$W_N(A) = p_N \ln(A + y_N) + q_N, \tag{7}$$

where $a_N = 1$, $p_N = \delta^{N-T}$, and $b_N = q_N = 0$. Using the forms deliberately suggested by (6) and (7), we conjecture that at any date $t < N$,

$$V_{t+1}(A) = a_{t+1} \ln(A + M_{t+1}) + b_{t+1},$$

and

$$W_{t+1}(A) = p_{t+1} \ln(A + M_{t+1}) + q_{t+1},$$

where the expression M_s simply records the present value of flow income from any date s; that is,

$$M_s = y_s + \frac{y_{s+1}}{1+r} + \cdots + \frac{y_N}{(1+r)^{N-s}}.$$

This conjecture helps us to set up the optimization problem (viewed from time 0) that will be solved at date t: for any asset level A at date t,

$$\max_c \ \alpha[\delta^t \ln c + \delta^{t+1} V_{t+1}(A')] + (1-\alpha)[\delta^{|t-T|} \ln c_+ W_{t+1}(A')],$$

where A' is just $(1+r)[A - c + y_t]$.

It is easy to see that the solution to this problem is given by

$$c = \frac{\alpha\delta^t + (1-\alpha)\delta^{|t-T|}}{\alpha\delta^t + (1-\alpha)\delta^{|t-T|} + \alpha a_{t+1}\delta^{t+1} + (1-\alpha)p_{t+1}}[A + M_t] \equiv \lambda_t[A + M_t]. \tag{8}$$

Using this rule, we obtain corresponding forms for V_t and W_t. First,

$$V_t(A) = a_t \ln(A + M_t) + b_t,$$

where

$$a_t = 1 + \delta a_{t+1} \tag{9}$$

and

$$b_t = \ln(\lambda_t) + \delta a_{t+1}[\ln(1+r) + \ln(1-\lambda_t)] + \delta b_{t+1}. \tag{10}$$

Similarly,

$$W_t(A) = p_t \ln(A + M_t) + q_t,$$

where

$$p_t = \delta^{|t-T|} + p_{t+1} \tag{11}$$

and

$$q_t = \delta^{|t-T|} \ln(\lambda_t) + p_{t+1}[\ln(1+r) + \ln(1-\lambda_t)] + q_{t+1}. \tag{12}$$

This completes (and justifies) the inductive step.

Equations (9) and (11) may be used to arrive at closed-form solutions for a_t and p_t. Doing so, we see that

$$a_t = \frac{1-\delta^{N-t+1}}{1-\delta}, \tag{13}$$

while

$$p_t = \frac{\delta[1-\delta^{T-t}]}{1-\delta} + \frac{1-\delta^{N-T+1}}{1-\delta} \text{ for } t < T, \tag{14}$$

$$= \frac{\delta^{t-T}[1-\delta^{N-t+1}]}{1-\delta} \text{ for } t \geq T. \tag{15}$$

Combining (8), (9), and (11), we can simplify the optimal consumption ratio λ_t to

$$\lambda_t = \frac{\alpha\delta^t + (1-\alpha)\delta^{|t-T|}}{\alpha\delta^t a_t + (1-\alpha)p_t}. \tag{16}$$

This is the expression that we are interested in evaluating, with the help of the closed forms (13) and (15). To this end, notice first that there is an intrinsic tendency for the consumption ratio to drift upwards simply by virtue of the finite horizon nature of the problem. For instance, in the last period, all of permanent income will be consumed. We can benchmark this drift by setting $\alpha = 1$ in the problem above, whereupon the situation reduces to a perfectly standard life cycle problem. The consumption ratio sequence for this problem, which we denote by $\bar{\lambda}_t$, can be separately computed or arrived at by putting $\alpha = 1$ in (16) above and then using (13):

$$\bar{\lambda}_t = \frac{1}{a_t} = \frac{1-\delta}{1-\delta^{N-t+1}}. \tag{17}$$

This benchmark helps us get a handle on the extent to which our model departs from the standard formulation (in terms of predicting additional savings out of permanent income). Using (16) and (17), form the ratio

$$\theta_t \equiv \frac{\lambda_t}{\bar{\lambda}_t} = \frac{\alpha + (1-\alpha)\delta^{|t-T|-t}}{\alpha + (1-\alpha)(p_t/a_t\delta^t)}, \tag{18}$$

and observe that a value of $\theta_t = 1$ implies that (at date t) there is no divergence between the consumption ratios predicted by the two models. On the other hand, if $\theta_t < 1$ then the model of parental influence predicts a higher savings rate, and this effect is directly related to the amount by which θ falls below unity.

To match the algebra with intuition, note that at any date $t \geq T$, there is no difference between the discounting exhibited by the current self and shadow parent. So consumption behavior should be quite independent of the weights placed on these two selves, and in particular should coincide with that of the standard model. Indeed, if we substitute the value of a_t (from (13) and the value of p_t (from (15) for $t \geq T$) in (18), we see that $\theta = 1$. For such time periods, there is no discrepancy (in consumption ratios) between our model and the standard formulation.

The more interesting comparison is for dates that are less than T. The following observation is critical: for all $0 \leq t < T$,

$$\delta^{T-t} < \frac{p_t}{a_t}. \tag{19}$$

This is easy enough to establish by direct computation, using (13) and (15).

Combining (19) with (18), it is easy to see that consumption ratios are lowered for all periods upto the age of the shadow parent. [Later, once we describe the equilibrium version, we shall attempt to provide quantitative estimates of the magnitude of this effect.]

It should also be clear that if the agent increases the weight on his shadow parent, then the savings rate increases at each date.

Finally, what is the relationship between the *extent* of divergence and time? At what point over the agent's lifetime do we observe maximal (planned) divergence from the standard model?

It turns out, not surprisingly, that the answer to this question depends on the weight that the agent attaches to the shadow parent. If this weight is high enough, the agent wants future versions of himself to carry out the bulk of the savings. Formally, the following is true:

Observation 1. *There exists $\hat{\alpha} \in (0, 1]$ such that if $\alpha \leq \hat{\alpha}$, θ_t always increases in t;* while if $\alpha > \hat{\alpha}$, *θ_t first decreases and then increases in t.*

The proof of this observation is relegated to the appendix. But a numerical example may be useful: say a person starts at age 30. If this is normalized to zero, we may take $N = 50$ and $T = 35$. If $\delta = 0.8$ and equal weight is put on the shadow parent so that $\alpha = 0.5$, it is easy to compute that θ_t begins at 0.36, reaches its minimum of 0.02 at $t = 16$, and goes back up to 1 at $t = 35$. For this value of δ, $\hat{\alpha} \approx 0.0005$. However, for $\delta = 0.95$, $\hat{\alpha} \approx 0.19$.

Notice that (apart from the intended illustration of Observation 1), the example shows that the effects on savings are extremely large. However, because these outcomes are not time-consistent, we return to the issue of magnitude when we study the equilibrium problem.

3.3.2 Equilibrium Consumption and Savings

Next, we take into account the fact that planned paths will not be honored by future generations. This is precisely an instance of the general problem posed by Strotz [1956], and analyzed by numerous authors (see, e.g., Phelps and Pollak [1968], Peleg and Yaari [1973], and Bernheim and Ray [1987]). The idea is to treat each generation as a separate player in a noncooperative game, and analyze the subgame perfect equilibria of such a game. It is well known from O'Donoghue and Rabin (1999) that people taking into consideration of this time-inconsistency (called sophisticates) could behave drastically different from people who do not (called naifs).

In the logarithmic case (and with constant elasticity utility functions more generally), we can solve this problem completely for the finite horizon case. It turns out that there is a unique subgame perfect equilibrium, which can be described using backwards recursion. We have already laid the groundwork for this in the previous section. To retain comparability, it will actually help to abuse notation and use exactly the same notation to describe value functions, consumption ratios, etc. We begin as before by observing that in the last period N, the value functions are exactly in the form given by (6) and (7), with the restrictions $a_N = 1$, $p_N = \delta^{N-T}$, and $b_N = q_N = 0$. Therefore conjecture that at some date t,

$$V_{t+1}(A) = a_{t+1} \ln(A + M_{t+1}) + b_{t+1},$$

and

$$W_{t+1}(A) = p_{t+1} \ln(A + M_{t+1}) + q_{t+1},$$

Now we can state the problem that faces the agent at date t: for any asset level A,

$$\max_c \quad \alpha[\ln c + \delta V_{t+1}(A')] + (1-\alpha)[\delta^{|t-T|} \ln c + W_{t+1}(A')],$$

where A' is $(1+r)[A-c+y]$, as before. Notice how in this problem the effective discount factors are different from those in the previous section. Solving this problem, we see that

$$c = \frac{\alpha + (1-\alpha)\delta^{|t-T|}}{\alpha + (1-\alpha)\delta^{|t-T|} + \alpha a_{t+1}\delta + (1-\alpha)p_{t+1}}[A + M_t] \equiv \lambda_t[A + M_t]. \tag{20}$$

This gives us solutions for V_t and W_t. First,

$$V_t(A) = a_t \ln(A + M_t) + b_t,$$

where

$$a_t = 1 + \delta a_{t+1} \tag{21}$$

and

$$b_t = \ln(\lambda_t) + \delta a_{t+1}[\ln(1+r) + \ln(1-\lambda_t)] + \delta b_{t+1}. \tag{22}$$

Similarly,

$$W_t(A) = p_t \ln(A + M_t) + q_t,$$

where

$$p_t = \delta^{|t-T|} + p_{t+1} \tag{23}$$

and

$$q_t = \delta^{|t-T|} \ln(\lambda_t) + p_{t+1}[\ln(1+r) + \ln(1-\lambda_t)] + q_{t+1}. \tag{24}$$

This completes the inductive step. Notice that these forms look no different from those in the previous problem (though the quantitative magnitudes of b_t and q_t will indeed be different because of the new values of λ_t for the equilibrium problem).

As before, (21) and (23) yield closed-form solutions for a_t and p_t:

$$a_t = \frac{1 - \delta^{N-t+1}}{1-\delta}, \tag{25}$$

and

$$\begin{aligned} p_t &= \frac{\delta[1-\delta^{T-t}]}{1-\delta} + \frac{1-\delta^{N-T+1}}{1-\delta} \text{ for } t < T, & (26) \\ &= \frac{\delta^{t-T}[1-\delta^{N-t+1}]}{1-\delta} \text{ for } t \geq T. & (27) \end{aligned}$$

Combining (20), (21), and (23), we can simplify the equilibrium consumption ratio λ_t to

$$\lambda_t = \frac{\alpha + (1-\alpha)\delta^{|t-T|}}{\alpha a_t + (1-\alpha)p_t}. \tag{28}$$

Notice that the solution to the a_t's and the p_t's are precisely what they were in the planned model (this gives the logarithmic case some extra sharpness). But the consumption ratios are different (compare (28) with its predecessor (16)). Recalling that consumption ratios in the standard model are given by $\bar{\lambda}_t = 1/a_t$, we can form our measure of relative consumption ratios, just as we did before, by

$$\theta_t \equiv \frac{\lambda_t}{\bar{\lambda}_t} = \frac{\alpha + (1-\alpha)\delta^{|t-T|}}{\alpha + (1-\alpha)(p_t/a_t)}, \tag{29}$$

Note that a value of $\theta_t = 1$ implies that (at date t) there is no divergence between the consumption ratios predicted by the two models. As in the case of planned consumption paths, it is easy to check that there is indeed no divergence between the two ratios when $t \geq T$.

For dates that are less than T, exactly the same observation as in (19) applies here: for all $0 \leq t < T$,

$$\delta^{T-t} < \frac{p_t}{a_t}. \tag{30}$$

Combining (30) with (29), it is easy to see that consumption ratios are lowered for all periods upto the age of the shadow parent. But we can say a bit more now than we could in the planning exercise: as long as $t < T$, *the extent of the divergence between equilibrium consumption ratios and the ratios in the standard model must monotonically decline over time.* That is, the greatest impetus to savings comes at the earliest dates (controlling for income, of course).

To check this, it is sufficient to see that δ^{T-t} increases over time, while p_t/a_t declines, and then apply these findings to (29).[5]

[5]It is immediate that δ^{T-t} increases with t for $t < T$. To check the claim for p_t/a_t, note that this

As we have argued (see the planned case), a monotonically decreasing divergence of consumption ratios is not necessarily what an agent might *want*. In general, he would like future incarnations of himself to do the bulk of the savings. In fact, it is easy to check that the divergence ratios are larger under the planning problem than under the equilibrium problem for every date $t < T$. But given time-inconsistency, he knows that these large desired divergences are not going to be honored. This realization is built in into the equilibrium problem.

We should note, however, that this result does *not* rule out a situation in which equilibrium savings rates rise and then fall over time. See, for example, Table 1 in the next section.

4 Discussion

It will be useful to summarize the foregoing analysis (including our findings in the planned case) in the form of a proposition:

PROPOSITION **2** *Suppose that an agent faces an accumulation problem, and places weights α on himself and $1-\alpha$ on a shadow parent of fixed age T. Then*

[1] *In both the planning and equilibrium versions of the problem, consumption ratios (out of permanent income) are lower relative to those obtained for the standard problem, at every date $t < T$.*

[2] *For dates $t \geq T$, there is no discrepancy between the consumption ratios.*

[3] *A larger weight on the shadow parent depresses consumption ratios even further (at each date), in both the planning and equilibrium versions.*

---

expression equals $\frac{\delta[1-\delta^{T-t}]+[1-\delta^{N-T+1}]}{1-\delta^{N-t+1}} \equiv \frac{\delta[1-x]+[1-D]}{1-Dx}$, where $x = \delta^{T-t}$ and $D = \delta^{N-T+1}$. It is easy to check (e.g., by differentiation) that this last expression is a monotonically decreasing function of x, while x is itself an increasing function of t.

Age	Benchmark	Weight on Shadow Parent				
	Model	0.1	0.2	0.3	0.4	0.5
30	**14**	**25**	**34**	**43**	**52**	**59**
40	**24**	**32**	**39**	**46**	**53**	**59**
50	**25**	**29**	**34**	**39**	**43**	**48**

Table 1: Savings Rates (%) in the Benchmark and Equilibrium Models.

[4] *The planned divergence ratios are larger (at each date t such that $0 < t < T$) than the equilibrium divergence ratios. Furthermore, the equilibrium divergence ratios monotonically decline over time, while no such presumption can be entertained for the planned divergence ratios (as described fully in Observation 1).*

Just how large are these effects? We have already seen a numerical example for the planned case, but we know that such planned divergences will not be observed. Hence it is the equilibrium case that merits a more careful study. Table 1 provides some numerical magnitudes for a particular parametric configuration. As our results are all analytical and as the numbers are quite robust to changes in the parameters, we did not feel it necessary to report on a wide variety of cases.

In what follows, we look at equilibrium savings rates for individuals of age 30, 40 and 50, when lifetime is taken to be 80 and the age of the shadow parent is set to a "retirement age" of 65. The interest rate on wealth is 7%, and we use a discount factor of 0.95. Our model individual receives a wage income of 1 unit per period until retirement, and begins life at age 30 with an asset level of 2 units.[6]

To facilitate the use of everyday empirical observations, the savings rates we report are *not* out of permanent income, but out of current income (which is wage income plus

[6] Alternative specifications are available on request from the authors.

any interest income on assets). The results are fairly strong. The first column of the table reports savings rates at various ages in a standard life-cycle "benchmark model" (which is just our model with α set equal to unity). The second column does the same for our equilibrium model when a low weight of 0.1 (that is, $\alpha = 0.9$) is assigned to the shadow parent. The effect on equilibrium savings, as Table 1 shows, is extremely strong. Savings rates nearly double at age 30 and the positive effects at later ages, while not as strong, are still significant.

As stated in Proposition 2, the bulk of the impact of parental influence takes place in the earlier phases of life (in the equilibrium model). This is clearly borne out by the table. Notice, however, that this does not preclude an increase in the savings rate *over time*, followed even by a decline. Under the weight of a large accumulation of assets, the natural tendency will be to curb savings somewhat in later years. This is obvious in the post-retirement phase, of course. But our numerical computations suggest that this phenomenon may be heightened (even at pre-retirement ages) in the presence of parental influence.

Finally, observe that further weight placed on the shadow parent lead to even more dramatic effects on the savings rate. These numbers are provided, not necessarily for the sake of realism, but to suggest that socio-cultural phenomena such as upbringing and influence may have effects on savings rates that simply swamp any changes that might be brought about by the usual economic policies.

5 On Backward Discounting

A basic premise of our model is that economic agents discount time both forward and backwards. The former phenomenon is familiar, and is often interpreted as the degree of impatience. The latter is less familiar, or at least little-used in economics. It may be interpreted as the weakness of memory.

It is obvious why backward discounting does not (and should not) occupy the same central location in economics that forward discounting does. The latter applies to decisions that are to be made; the former (often) applies to the evaluation of choices which have been made in the past. The past cannot be changed, so that backward-discounting is accordingly less central in the theory of decision-making.

This argument is problematic from two points of view. First, even if the past cannot be changed, it still affects perceptions of happiness. Backward discounting suggests that the past will always be viewed through the opposite end of the telescope. This suggests that feelings of regret may be more endemic than we think. Indeed, if this subsequent view from the "opposite end" is not fully internalized at the time of decision-making, the model consistently *predicts* regret — an observation that may be of some use in psychology.

Second, future valuations of experience may influence the way in which we make current decisions. When we explore the implications of this hypothesis, the issue of backward discounting acquires new importance.

There are several situations in which "backward valuations" might influence "forward decisions". This paper is about one such situation, in which a future self's utility from a sequence of decisions is taken into accounting, knowing that this future self will be looking back on these decisions, and hence discounting backward. But there are other situations, and the purpose of this section is to summarize some of them.

5.1 Social Decision-Making

As Caplin and Leahy [1999] have observed, the phenomenon of backward discounting can give rise quite naturally to an argument for greater patience on the part of a social planner, compared to the discount factor of the agents in the economy. Suppose that generation τ derives lifetime utility V_τ from a consumption stream $\{c_t\}$. A social planner might want to choose $\{c_t\}$ to maximize some weighted sum of these generational utilities,

$$\lambda_\tau V_\tau,$$

where the λ's are positive weights that sum to unity. Assume that each generation exhibits a mixture of backward and forward discounting, so that:

$$V_\tau(\{c_t\} = \sum_{s=\tau}^{\infty} \beta_1^{s-\tau} u(c_s) + \sum_{s=0}^{\tau-1} \beta_2^{\tau-s} u(c_s),$$

where β_1 is the foward rate and β_2 is the backward rate. The it is easy to see that the planner will exhibit a degree of patience that exceeds the patience implied in β_1. [To be sure, there are time-consistency problems as well, but that isn't the main point here.]

5.2 Case-Based Decision Theory

Gilboa and Schmeidler [1995] have introduced case-based decision theory, in which new situations of decision-making must be studied by studying parallels within the realm of one's own experience. They propose a framework in which such parallel cases are taken as primitives, and seek to derive (axiomatically) decision rules which prescribe a best course of action depending on past performance in parallel cases. They note that both the degree of "similarity" to the case at hand, as well as the utility derived from the similar case, play a role in this axiomatization.

This is a setting in which — for decision problems that involve time — backward discounting arises in a natural measure of "utility from parallel cases". Because *past* experience is used as a guide to decision-making *for the future*, case-based decision theory is a natural setting for backward discounting to play a prominent role.

One implication of this kind of reasoning is that we may actually behave more patiently in real life than our "natural inclinations" would have us behave, an observation that goes well with the rest of the theme of this paper.

5.3 Evolutionary Models

The last observation also applies to evolutionary situations in which "success" or "fitness" is measured by some discounted sum of intertemporal rewards, where measurement takes place at the "end of the day". This might be particularly apt in the context of evolutionary biology, in which fitness (over some intertemporal setting) is determined by the *end-state* value of some variable. Concentration on such end-state outcomes is equivalent to a very strong form of backward discounting in which the discount factor is set equal to zero.

Alternatively, in social or cultural contexts, "success" may be defined by the perceived lifetime rewards to an individual (or a role model, or a way of life) *at the end of the process*. If these perceptions form the basis for cultural selection, then agents (or behavior patterns) that assign heavier weight to future consumptions would be the winners; in other words, agents (or cultural modes) that promote harder work and more savings when young are more likely to survive.

To be sure, there is no guarantee of such an outcome. It would all depend on which age groups (or which life stages) are considered natural points of success evaluation. These "natural" points are themselves subject to evolutionary or cultural pressures, leading to a deeper level of recursive analysis that is beyond the scope of this paper.

5.4 Elections

We end with a provocative example, not one we necessarily believe in fully, but one that is interesting and far from being obviously false. Consider elections and reelections.

Suppose that a person can stay in office for at most two terms, and each term lasts for N periods. To simplify matters, assume that an elected officer's task is to provide a plan of allocating consumptions over these N periods and commit to it. Consider the first term. Let the total assets be A, the interest rate be r, c_i, $i = 1, ..., N$, be period

i's consumption, and $u(\cdot)$ be a concave utility function. Let $V = \sum_{n=1}^{N} \delta^{n-1} u(c_n)$ be the conventionally discounted total utility. A candidate's probability of being elected is then a function of V.

It is reasonable to assume that the probability of being re-elected for a second term depends on the incumbent's history. At the re-election date, voters may recall their past consumptions using backward discounting. Let $V_b = \sum_{n=1}^{N} \delta^{N-n} u(c_n)$ be the backward discounted total utility of the first term. A candidate then maximizes the probability of elected and re-elected given by the hypothetical function $P(V, V_b)$, which is increasing in both of its arguments. For simplicity, assume that $\delta = \frac{1}{1+r}$.

It is straight-forward to verify that $c_1 = \cdots = c_N = A/(1+\delta+\cdots+\delta^{N-1})$ maximizes V, subject to budget constraint $\sum_{n=1}^{N} \delta^{n-1} c_n = A$. That is, if a candidate does not aim for re-election, or, if the voters discount utilities in the conventional way, a plan of constant consumption over time would be proposed.

Maximizing $P(V, V_b)$ subject to the same budget constraint implies the following equation: $P_1 \delta^{n-1} u'(c_n) + P_2 \delta^{N-n} u'(c_n) = \lambda \delta^{n-1}$, where $P_1 = \frac{\partial P(V,V_b)}{\partial V} > 0$, $P_2 = \frac{\partial P(V,V_b)}{\partial V_b} > 0$, and λ is the Lagrange multiplier. Therefore,

$$u'(c_n) = \frac{\lambda}{P_1 + P_2 \delta^{N-2n+1}}.$$

As the right hand side of the above equation is decreasing in n, and $u(\cdot)$ is a concave utility function, we have the following relationship: $c_1 < c_2 < \cdots < c_N$. That is, consumptions are increasing over time, since later consumptions matter more to the voters at time of re-election.

If an elected candidate does not need to keep his promise, he would propose a plan of constant consumption to maximize V, and thus the probability of being elected. But once elected, he would maximize V_b by reducing the consumptions of early periods and increasing the consumptions of later periods, so that his probability of being re-elected is maximized. The exact numbers can be calculated by setting $P_1 = 0$ in the above derivations. We can easily see that it makes the consumption profile even steeper, and thus reinforces the effects of re-election. This cannot happen in a model with conventional discounting.

6 Summary

In this paper, we introduce a framework in which lifetime individual felicities are derived from both present and past consumption streams. Each of these streams is discounted, the former forward in the usual way, the latter backward. *Parental influence* refers to a state of affairs in which an individual at date t evaluates consumption programs according to some weighted average of his own felicity (as perceived at date t) and that of a "shadow parent" at some date $T > t$). This simple model can be used, among other things, to show that parental influence may have a positive impact on savings, that individuals may exhibit impatience across alternatives that are positioned in periods adjacent to the present, but patience across similar choices positioned in the more distant future, that such impatience is attenuated as an individual grows older, and that lifetime choice plans are generally time-inconsistent. The postulate of "backwards discounting" used in the model may also be of intrinsic interest, and we discuss this by means of several additional examples.

7 Appendix

Here is the proof of Observation 1. Let $D = \delta^{N-T+1}$, $x(t) = \delta^{T-t}$, $f(x) = x^2/\delta^T$, and

$$g(x) = \frac{[\delta(1-x)+1-D]x}{(1-Dx)\delta^T} = \frac{x}{D\delta^{T-1}} + \frac{(1-D+\delta-\frac{\delta}{D})x}{(1-Dx)\delta^T}.$$

Then

$$\theta_t = \frac{\alpha+(1-\alpha)f(x(t))}{\alpha+(1-\alpha)g(x(t))} \equiv h(x(t)).$$

It is straight-forward to verify that $1-D+\delta-\frac{\delta}{D}=(1-\delta^{N-T})(\delta-\delta^{T-N})<0$. Therefore, $g''(x)<0$. Furthermore, $h'(x)=0$ implies that $f'(x)/g'(x)=h(x)$, which in turn implies that $g'(x)>0$ at $h'(x)=0$. Making use of all of the above, and evaluating $h''(x)$ at $h'(x)=0$, we obtain $h''(x)>0$. This indicates that $h(x)$ has at most one stationary point, and if that exists, $h(x)$ is decreasing on the left-hand side of that point and increasing on the right-hannd side.

Tedious calculations show that at $x=\delta^T$ (i.e., at $t=0$), $f(x)<g(x)$; at $x=1$ (i.e., at $t=T$), $f(x)=g(x)$. Therefore, $h(x(0))<h(x(T))=1$. Furthermore, $f'(x)g(x)-f(x)g'(x)>0$ at $x=\delta^T$. Hence,

$$h'(x)=\frac{1-\alpha}{[\alpha+(1-\alpha)g(x)]^2}\{f'(x)g(x)-f(x)g'(x)+\alpha[f'(x)-g'(x)-f'(x)g(x)+f(x)g'(x)]\}$$

is positive at $\alpha=0$, and possibly turns negative for larger α's if $f'(x)-g'(x)-f'(x)g(x)+f(x)g'(x)<0$. The initial slope of $h(x)$ establishes whether $h(x)$ is U-shaped or always increasing. If the initial slope is negative, then it is isn U-shaped. Otherwise, it is always increasing.

Since $x=x(t)$ is an increasing function of t, the same property holds for θ_t. ■

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