Debraj Ray

Income Distribution and Macroeconomic Behavior

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There are an infinite number of generations, one at each date. In each period t a member of generation receives an income w, which depends on the proportions of the population in different skill categories, as well as the skill category to which this member belongs. This income is partly consumed and partly used in educating the offspring of this member. Depending on the education level of the child, the child receives an income next period, and the entire process repeats itself.

At each date, a production function f determines the wage to skill categories. Assume that there are only two skill categories: "high" and "low". Being low at any date requires no investment by the parent; being high requires an investment of x, which is denominated in terms of final output and kept fixed for the purpose of the exercise. If there is a unit mass of individuals, and a fraction λ of them is high at some date, then the high wage is given by

$$\bar{w}(\lambda) \equiv f_1(\lambda, 1-\lambda),$$

while the low wage is given by

$$\underline{w}(\lambda) \equiv f_2(\lambda, 1-\lambda).$$

If f satisfies the usual curvature and end-point conditions, we can conclude that $\bar{w}(\lambda)$ is decreasing and continuous in λ, with $\bar{w}(\lambda) \to \infty$ as $\lambda \to 0$. Likewise, $\underline{w}(\lambda)$ is increasing and continuous in λ, with $\underline{w}(\lambda) \to \infty$ as $\lambda \to 1$. We may thus define a threshold $\tilde{\lambda}$ such that $\bar{w}(\tilde{\lambda}) = \underline{w}(\tilde{\lambda})$.

Imagine, now, that an infinite sequence of wages is given, one of each category of labor. We may denote this by the path $\{\bar{w}_t, \underline{w}_t\}_{t=0}^{\infty}$. With such a sequence given, we take it that generation t maximizes an additive function of the utility from its own consumption, and the (discounted) utility felt by generation $t+1$. As in the Loury model, it is needed to conjecture what these future utilities might be, and standard dynamic programming arguments will tell us that there is a unique solution to this problem provided that we consider bounded wage paths. This unique solution is actually a utility *path*, which makes it a bit different from the Loury model, in which the environment is stationary by assumption. This path may be described as the sequence $\{\bar{V}_t, \underline{V}_t\}_{t=0}^{\infty}$, and it has the property that for each date t

$$\bar{V}_t = \max u(c_t) + \delta V_{t+1}$$

subject to the conditions that

$$c_t + x_t = \bar{w}_t,$$

and

$$\begin{aligned} V_{t+1} &= \bar{V}_{t+1} \text{ if } x_t \geq x \\ &= \underline{V}_{t+1} \text{ if } x_t < x. \end{aligned}$$

Likewise,

$$\underline{V}_t = \max u(c_t) + \delta V_{t+1}$$

subject to the conditions that

$$c_t + x_t = \underline{w}_t,$$

and

$$\begin{aligned} V_{t+1} &= \bar{V}_{t+1} \text{ if } x_t \geq x \\ &= \underline{V}_{t+1} \text{ if } x_t < x. \end{aligned}$$

Note: we are defining utility on the entire real line, instead of just the nonnegative numbers. This captures the idea that the borrowing constraint is not total, but that the investment of x at lower wealth levels carries greater utility losses.

These maximization problems describe how the education levels change from generation to generation, given the sequence of wage rates. In a general equilibrium setting, the opposite implication needs to be considered as well: given the skill choices made for generation $t+1$ by their parents, wage rates are determined for generation $t+1$. Of course, no individual can internalize this causal relationship, being infinitesimally small relative to the economy.

For given $\lambda_0 \in (0,1)$, a *competitive equilibrium* is a sequence $\{\bar{w}_t, \underline{w}_t \lambda_t\}_{t=0}^{\infty}$ such that

[i] Given λ_0, the path $\{\lambda_t\}$ is generated by the maximization problems just described,

[ii] For each t, $\bar{w}_t = \bar{w}(\lambda_t)$ and $\underline{w}_t = \underline{w}(\lambda_t)$ if $\lambda_t < \tilde{\lambda}$, and $\bar{w}_t = \underline{w}_t = \bar{w}(\tilde{\lambda}) = \underline{w}(\tilde{\lambda})$ if $\lambda_t \geq \tilde{\lambda}$.

Note that the definition of a competitive equilibrium uses the idea that if the "natural wages" (as given by marginal product) for a skilled worker falls short of that of an unskilled worker, the former will move intro the sector of the latter so that the two wages will be *ex post* equalized. In any case this is not very relevant because of the following trivial observation:

LEMMA **1** *Along any equilibrium,* $0 < \lambda_t < \tilde{\lambda}$ *for all* $t \geq 1$.

The proof of this lemma should be obvious. That $\lambda_t > 0$ for all t follows from the fact that the difference between skilled and unskilled wages would be infinitely high otherwise, so that some educational investment would have taken place prior to that period. On the other hand, λ_t cannot exceed $\tilde{\lambda}$ for any $t \geq 1$, for in that case high and low wages are equalized. Who in the previous generation would have invested in such circumstances?

From now on I will also assume that $\lambda_0 \in (0, \tilde{\lambda})$ as well. There is no big mystery in making this assumption: it saves having to qualify statements for the initial value of λ.

LEMMA **2** *Under a competitive equilibrium, there is no date at which a low person creates a high kid while simultaneously, a high person creates a low kid.*

Proof. If a low person creates a high kid, then

$$u(\underline{w}_t - x) + \delta \bar{V}_{t+1} \geq u(\underline{w}_t) + \delta \underline{V}_{t+1},$$

or

$$u(\underline{w}_t) - u(\underline{w}_t - x) \leq \delta[\bar{V}_{t+1} - \underline{V}_{t+1}].$$

By strict concavity and the fact that $\lambda_t < \tilde{\lambda}$ for all t, we may conclude that

$$u(\bar{w}_t) - u(\bar{w}_t - x) < \delta[\bar{V}_{t+1} - \underline{V}_{t+1}].$$

But this means that a high person has a *strict* incentive to create a high kid, and we are done. ■

A fraction λ is called a *steady state* if there exists a competitive equilibrium from λ with $\lambda_t = \lambda$ for all t.

Let $\bar{w} \equiv \bar{w}(\lambda)$ and $\underline{w} \equiv \underline{w}(\lambda)$. Note that for the condition above of a steady state to be satisfied, it is necessary and sufficient that

$$\bar{V} = u(\bar{w} - x) + \delta\bar{V} \geq u(\bar{w}) + \delta\underline{V},$$

while

$$\underline{V} = u(\underline{w}) + \delta\underline{V} \geq u(\underline{w} - x) + \delta\bar{V}.$$

Combining these two expressions, we may conclude that

$$u(\bar{w}) - u(\bar{w} - x) \leq \delta(\bar{V} - \underline{V}) \leq u(\underline{w}) - u(\underline{w} - x)$$

is a necessary and sufficient condition for λ to be a steady state. Combining this expression for the values of $\bar{V}$ and $\underline{V}$, we see that we have established

Proposition **1** *The fraction λ is a steady state if and only if*

$$u(\bar{w}) - u(\bar{w} - x) \leq \frac{\delta}{1-\delta}[u(\bar{w} - x) - u(\underline{w})] \leq u(\underline{w}) - u(\underline{w} - x) \tag{1}$$

Equivalently, the following pair of conditions must hold:

$$u(\bar{w}) - u(\bar{w} - x) \leq \delta[u(\bar{w}) - u(\underline{w})] \tag{2}$$

and

$$u(\underline{w}) - u(\underline{w} - x) \geq \delta[u(\bar{w} - x) - u(\underline{w} - x)] \tag{3}$$

What does the set of steady states look like? Note that as $\lambda \to \tilde{\lambda}$, the middle term of (1) is negative while the left term is positive. On the other hand, as $\lambda \to 0$, the middle term goes to infinity while the left term is bounded above. Because the changes are monotone, there is a unique $\lambda^* \in (0, \tilde{\lambda})$ such that the first inequality in (1) holds with equality. Observe, moreover, that at $\lambda = \lambda^*$, the *second* inequality in (1) must hold as well, because of the strict concavity of the utility function. Thus the set of steady states contains some interval to the left of λ^*, and must be a subset of $(0, \lambda^*]$. But beyond this nothing much more can be said about the set of steady states. In fact, it need not be connected (why?).

Now we begin the study of non-steady-state dynamics. Our objective will be to prove that starting from any initial fraction of high people, any equilibrium path must converge to some steady state. In particular, even if we start from a distribution of income that is equal, the final distribution of income must involve inequality.

The following simple lemma is crucial to all that follows. It states that at every date, the lifetime utility of the high (and low) must be equal to the utility they would have received were their descendants *never* to switch status. [Note that along the equilibrium path, switching of status will generally occur, however.]

LEMMA **3** *Let* $\{\bar{w}_t, \underline{w}_t, \lambda_t\}_{t=0}^{\infty}$ *be a competitive equilibrium. Then for each date* t*,*

$$\bar{V}_t = \sum_{s=t}^{\infty} \delta^{s-t} u(\bar{w}_s - x) \tag{4}$$

and

$$\underline{V}_t = \sum_{s=t}^{\infty} \delta^{s-t} u(\underline{w}_s). \tag{5}$$

Proof. It suffices to show that for each $t \geq 0$,

$$\bar{V}_t = u(\bar{w}_t - x) + \delta \bar{V}_{t+1}$$

and

$$\underline{V}_t = u(\underline{w}_t) + \delta \underline{V}_{t+1}.$$

To prove this, apply Lemmas 1 and 2. By Lemma 1 and our restriction on λ_0, $\lambda_t \in (0, \tilde{\lambda})$ for all $t \geq 0$. Now using Lemma 2, we may conclude that at all dates, some of the high people stay high, while some of the low people stay low. This is enough to establish the result. ■

Thus along any competitive equilibrium, no person will ever want to strictly change his actions, though it may well be the case that he strictly prefers to stay where he is. We may use Lemma 3 to write this condition as

$$\sum_{s=t}^{\infty} \delta^{s-t} u(\bar{w}_s - x) \geq u(\bar{w}_t) + \sum_{s=t+1}^{\infty} \delta^{s-t} u(\underline{w}_s),$$

or equivalently, as

$$u(\bar{w}_t) - u(\bar{w}_t - x) \leq \sum_{s=t+1}^{\infty} \delta^{s-t} [u(\bar{w}_s - x) - u(\underline{w}_s)], \tag{6}$$

with equality holding whenever a switch from "high" to "low" does occur along the equilibrium path. Likewise, we see that for the currently low,

$$\sum_{s=t}^{\infty} \delta^{s-t} u(\underline{w}_s) \geq u(\underline{w}_t - x) + \sum_{s=t+1}^{\infty} \delta^{s-t} u(\bar{w}_s - x),$$

or equivalently,

$$u(\underline{w}_t) - u(\underline{w}_t - x) \geq \sum_{s=t+1}^{\infty} \delta^{s-t} [u(\bar{w}_s - x) - u(\underline{w}_s)], \tag{7}$$

with equality holding whenever a switch from "low" to "high" does occur along the equilibrium path.

To proceed further, it will be necessary to consider two possible zones in which λ might lie, when λ is *not* a steady state. We divide the non-steady state space into two complementary parts: the first subset, which we denote by A, is the one in which the first inequality of (1), or equivalently, (2), fails. The second subset, which we denote by B, is the one in which the second inequality of (1), or equivalently, (3), fails. Note that A and B are disjoint because by the strict concavity of u, both these inequalities cannot *simultaneously* fail. Indeed, using the construction of the steady state set described earlier, it should be clear that $A = (\lambda^*, \tilde{\lambda})$ (assuming that we restrict attention to $\lambda \in (0, \tilde{\lambda})$), and B is some subset of the interval $(0, \lambda^*)$.

In what follows, we relate the dynamics of λ to membership in one of the sets A and B.

LEMMA **4** *If $\lambda_t > \lambda_{t+1}$, then $\lambda_t \in A$ and $\lambda_{t+1} = \lambda_{t+2}$.*
If $\lambda_t < \lambda_{t+1}$, then $\lambda_t \in B$ and $\lambda_{t+1} \leq \lambda_{t+2}$.

Warning. Note that the two statements in the lemma are *not* symmetric. The lack of symmetry will become even clearer later.

Proof of Lemma 4. *We begin by establishing the first part of the first statement.* Because $\lambda_t > \lambda_{t+1}$, (6) must hold with equality, and we have

$$u(\bar{w}_t) - u(\bar{w}_t - x) = \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1})] + \delta^2 M \tag{8}$$

where $M \equiv \sum_{s=t+2}^{\infty} \delta^{s-(t+2)}[u(\bar{w}_s - x) - u(\underline{w}_s)]$. Using (6) for period $t+1$, we see that

$$u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x) \leq \delta M. \tag{9}$$

Combining (8) and (9), we see that

$$u(\bar{w}_t) - u(\bar{w}_t - x) \geq \delta[u(\bar{w}_{t+1}) - u(\underline{w}_{t+1})].$$

Because $\lambda_t > \lambda_{t+1}$, we see that $\bar{w}_{t+1} > \bar{w}_t$ and $\underline{w}_{t+1} < \underline{w}_t$. Therefore

$$u(\bar{w}_t) - u(\bar{w}_t - x) > \delta[u(\bar{w}_t) - u(\underline{w}_t)],$$

which shows that $\lambda_t \in A$.

The proof of the first part of the second statement is completely parallel, but because (as noted above) there is an asymmetry lurking here it will be useful to simply retrace these steps and convince ourselves that they indeed go through.

For this part, $\lambda_t < \lambda_{t+1}$, so that (7) must hold with equality, and we have

$$u(\underline{w}_t) - u(\underline{w}_t - x) = \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1})] + \delta^2 M \tag{10}$$

where M is defined just as before. Using (7) for period $t+1$, we see that

$$u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x) \geq \delta M. \tag{11}$$

Combining (10) and (11), we see that

$$u(\underline{w}_t) - u(\underline{w}_t - x) \leq \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1} - x)].$$

Because $\lambda_t < \lambda_{t+1}$, we see that $\bar{w}_{t+1} < \bar{w}_t$ and $\underline{w}_{t+1} > \underline{w}_t$. Therefore

$$u(\underline{w}_t) - u(\underline{w}_t - x) < \delta[u(\bar{w}_t - x) - u(\underline{w}_t - x)],$$

which shows that $\lambda_t \in B$.

Next, we establish the second part of the first statement: that $\lambda_{t+1} = \lambda_{t+2}$. Suppose this is false. Then there are two cases to consider.

CASE 1: $\lambda_{t+1} < \lambda_{t+2}$. Then at date $t+1$, (7) must hold with equality, so that

$$u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x) = \delta M. \tag{12}$$

Combining (8) and (12), we see that

$$u(\bar{w}_t) - u(\bar{w}_t - x) = \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1} - x)]. \tag{13}$$

Because $\lambda_t > \lambda_{t+1}$, we have $\bar{w}_t > \underline{w}_t > \underline{w}_{t+1}$. Consequently, by the strict concavity of the utility function,

$$u(\bar{w}_t) - u(\bar{w}_t - x) < u(\underline{w}_t) - u(\underline{w}_t - x) < u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x). \tag{14}$$

Combining (13) and (14), we may conclude that

$$u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x) > \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1} - x)].$$

But this means that λ_{t+1} satisfies (3), or equivalently, that $\lambda_{t+1} \notin B$. On the other hand, we have $\lambda_{t+1} < \lambda_{t+2}$, and this contradicts the first part of the second statement of the lemma, which we have already proved.

CASE 2: $\lambda_{t+1} > \lambda_{t+2}$. Then at date $t+1$, (6) must hold with equality, so that

$$u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x) = \delta M. \tag{15}$$

Combining (8) and (15), we see that

$$u(\bar{w}_t) - u(\bar{w}_t - x) = \delta[u(\bar{w}_{t+1}) - u(\underline{w}_{t+1})]. \tag{16}$$

Because $\lambda_t > \lambda_{t+1}$, we have $\bar{w}_t < \bar{w}_{t+1}$. Consequently, by the strict concavity of the utility function,

$$u(\bar{w}_t) - u(\bar{w}_t - x) > u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x). \tag{17}$$

Combining (16) and (17), we may conclude that

$$u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x) < \delta[u(\bar{w}_{t+1}) - u(\underline{w}_{t+1})].$$

But this means that λ_{t+1} satisfies (2), or equivalently, that $\lambda_{t+1} \notin A$. On the other hand, we have $\lambda_{t+1} > \lambda_{t+2}$, and this contradicts the first part of the first statement of the lemma, which we have already proved.

Finally, we prove the second part of the second statement: that $\lambda_{t+1} \leq \lambda_{t+2}$. Suppose this is false. Then $\lambda_{t+1} > \lambda_{t+2}$. Thus at date $t+1$, (6) must hold with equality, so that

$$u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x) = \delta M. \tag{18}$$

Combining (10) and (18), we see that

$$u(\underline{w}_t) - u(\underline{w}_t - x) = \delta[u(\bar{w}_{t+1}) - u(\underline{w}_{t+1})]. \tag{19}$$

Because $\lambda_t < \lambda_{t+1}$, we have $\underline{w}_t < \underline{w}_{t+1} \leq \bar{w}_{t+1}$. Consequently, by the strict concavity of the utility function,

$$u(\underline{w}_t) - u(\underline{w}_t - x) > u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x) \geq u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x). \tag{20}$$

Combining (19) and (20), we may conclude that

$$u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x) < \delta[u(\bar{w}_{t+1}) - u(\underline{w}_{t+1})].$$

But this means that λ_{t+1} satisfies (2), or equivalently, that $\lambda_{t+1} \notin A$. On the other hand, we have $\lambda_{t+1} > \lambda_{t+2}$, and this contradicts the first part of the first statement of the lemma, which we have already proved.

■

Lemma **5** *If λ is a steady state, then there is a unique competitive equilibrium from $\lambda_0 = \lambda$, given by $\lambda_t = \lambda$ for all t.*

Proof. Immediate from Lemma 4. For if the competitive equilibrium is nonstationary, then it must be the case that either $\lambda \in A$ or $\lambda \in B$ (simply examine the first date that $\lambda_t \neq \lambda_{t+1}$ and apply Lemma 4). In either of these cases, λ cannot be a steady state. ■

A converse to this result is the subject of the next lemma.

Lemma **6** *If at any date t along a competitive equilibrium we have $\lambda_t = \lambda_{t+1}$, then $\lambda \equiv \lambda_t = \lambda_{t+1}$ is a steady state, and in particular $\lambda_s = \lambda_t$ for all $s \geq t$.*

Proof. Suppose not. Then by Lemma 5, it must be the case that either $\lambda \in A$ or $\lambda \in B$.

Case 1: $\lambda \in A$. In this case, renumbering time periods if necessary, we must have $\lambda_t = \lambda_{t+1} > \lambda_{t+2}$ (using Lemma 4). Thus (6) must hold with equality at date $t+1$, so that

$$u(\bar{w}_{t+1}) - u(\bar{w}_{t+1} - x) = \delta M, \tag{21}$$

while at date t

$$u(\bar{w}_t) - u(\bar{w}_t - x) \leq \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1})] + \delta^2 M \tag{22}$$

Combining (21) and (22), we see that

$$\begin{aligned} u(\bar{w}_t) - u(\bar{w}_t - x) &\leq \delta[u(\bar{w}_{t+1}) - u(\underline{w}_{t+1})] \\ &= \delta[u(\bar{w}_t) - u(\underline{w}_t)]. \end{aligned}$$

But this means that $\lambda \notin A$, which is a contradiction.

Case 2: $\lambda \in B$. In this case, renumbering time periods if necessary, we must have $\lambda_t = \lambda_{t+1} < \lambda_{t+2}$ (using Lemma 4). Thus (7) must hold with equality at date $t+1$, so that

$$u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x) = \delta M, \tag{23}$$

while at date t

$$u(\underline{w}_t) - u(\underline{w}_t - x) \geq \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1})] + \delta^2 M \tag{24}$$

Combining (23) and (24), we see that

$$\begin{aligned} u(\underline{w}_t) - u(\underline{w}_t - x) &\geq \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1} - x)] \\ &= \delta[u(\bar{w}_t - x) - u(\underline{w}_t - x)]. \end{aligned}$$

But this means that $\lambda \notin B$, which is a contradiction.

So neither Case 1 nor Case 2 is possible. This means that λ is a steady state. Applying Lemma 5, we see that there is a unique stationary equilibrium, and we are done. ∎

We may use these lemmas to arrive at our main theorem.

Proposition **2** *If $\lambda \in A$, then there exists a unique competitive equilibrium from λ which goes to the steady state in one period: $\lambda = \lambda_0 > \lambda_1 = \lambda_t$ for all $t \geq 1$.*

If $\lambda \in B$, then there exists a unique competitive equilibrium in which the proportion of high people increases strictly in every period, and converges to some steady state: $\lambda_t < \lambda_{t+1}$ for all $t \geq 0$.

If λ is a steady state, then there is a unique competitive equilibrium from $\lambda_0 = \lambda$, given by $\lambda_t = \lambda$ for all t.

Proof. To prove the first part of the proposition, note that *if* there is a competitive equilibrium, then by Lemmas 4 and 6, it must have the property discussed in the statement of the proposition. To check existence and uniqueness, define λ_1 by

$$u(\bar{w}(\lambda)) - u(\bar{w}(\lambda) - x) \equiv \delta(1-\delta)^{-1}[u(\bar{w}(\lambda_1) - x) - u(\underline{w}(\lambda_1))].$$

It is easy to see that λ_1 is well-defined and unique, and that $\lambda_1 < \lambda$. Now check that this gives us a competitive equilibrium, and that there is no other way of constructing an path that satisfies both (6) and (7).

To prove the second part of the proposition, we first need to strengthen the implication of Lemma 4 in this case. It will be enough to strengthen the second part of the statement of that lemma to: If $\lambda_t < \lambda_{t+1}$, then $\lambda_t \in B$ and $\lambda_{t+1} < \lambda_{t+2}$.

All of this is proved except for the stronger implication: $\lambda_{t+1} < \lambda_{t+2}$. To establish this, suppose that the assertion is false. Then, using Lemma 4, it must be the case that $\lambda_t < \lambda_{t+1} = \lambda_{t+2}$. By Lemma 6, we have $\lambda_{t+1} = \lambda_s$ for all $s \geq t+1$. Also, (7) must hold with equality at date t. Combining these two pieces of information, we see that

$$u(\underline{w}_t) - u(\underline{w}_t - x) = \delta(1-\delta)^{-1}[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1})].$$

Now $\lambda_t < \lambda_{t+1}$, so that $\underline{w}_t < \underline{w}_{t+1}$. By the strict concavity of u and the equality above,

$$u(\underline{w}_{t+1}) - u(\underline{w}_{t+1} - x) < \delta(1-\delta)^{-1}[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1})].$$

But this means that $\lambda_{t+1} \in B$ as well. But then by Lemma 6, it cannot be the case that $\lambda_{t+1} = \lambda_{t+2}$.

To prove existence and uniqueness from this initial condition, define recursively for each λ_t, the value of λ_{t+1} that solves the equation

$$u(\underline{w}_t) - u(\underline{w}_t - x) \equiv \delta[u(\bar{w}_{t+1} - x) - u(\underline{w}_{t+1} - x)], \tag{25}$$

where $\bar{w}_{t+1}$ and $\underline{w}_{t+1}$ are to be interpreted as the wages corresponding to λ_{t+1}.

To see that this is uniquely defined, note that

$$u(\underline{w}_0) - u(\underline{w}_0 - x) < \delta[u(\bar{w}_0 - x) - u(\underline{w}_0 - x)],$$

because $\lambda_0 \in B$. So there is a unique λ_1 that solves (25) for $t = 0$. Note that λ_1 must exceed λ_0. And this will be so whenever $\lambda_t \in B$. So it only remains to show that if $\lambda_t \in B$, then $\lambda_{t+1} \in B$ as well. To see thus simply use the fact that $\lambda_{t+1} > \lambda_t$, which implies that $\underline{w}_{t+1} > \underline{w}_t$. Using this information in (25) along with the strict concavity of u, we are done.

The trick to understanding that this is the *only* way to construct a competitive equilibrium from this initial condition, because (7) will have to hold with equality.

Finally, part 3 of the proposition is already established. ■