Apples and Oranges: Building a Better Dow

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1. Introduction

The Dow Jones Industrial Average is among the most closely watched stock market indexes in the world. It is also an enormously valuable resource for research, with an uninterrupted data series stretching back to 1896. Yet it has some glaring omissions. Apple, currently the largest publicly traded company in the world with a market capitalization of over $577 billion, isn't a Dow component. Nor is Google, with a higher valuation than all but a handful of firms in the index. Meanwhile, companies with less than a tenth of Apple's valuation, including Alcoa and Hewlett-Packard, continue to be included.

The reason for such seemingly arbitrary omissions has to do with the manner in which the Dow is computed. It is a price-weighted index, and the average price of its thirty components is currently around $58. Both Apple and Google have share prices in excess of $600, and their inclusion would cause day-to-day changes in the index to be driven largely by the behavior of these two securities. As we show below, their combined weight in the Dow would be about 43% if they were both to be included in place of the two current components (Alcoa and Travelers) with the lowest market capitalizations. Furthermore, the index would become considerably more volatile even if the included stocks were individually no more volatile than those they replace. As John Prestbo, Chairman of the Index Oversight Committee recently observed, such heavy dependence of the index on one or two stocks would "hamper its ability to accurately reflect the broader market."[1]

Indeed, price-weighting is decidedly an odd methodology. Simply adding Chevron's share price to that of AT&T (as the Dow does) ensures that a one-dollar change in the price of Chevron will have the same effect on the index as a one-dollar change in the price of AT&T. But a one-dollar change works out to well under a 1% change in the share price of Chevron (0.89% as of August 3, 2012), while that same dollar change means a 2.7% change in the share price of AT&T. That means Chevron is *three times as important* in affecting the Dow as AT&T is, and for no legitimate reason. The market capitalization of each company is just a shade over $218b; they are equally valuable.

[1]John Prestbo, "Why AAPL and GOOG Aren't in the Dow." *Indexology*, February 8 2012.

This issue does not arise with value-weighted indexes such as the S&P 500. But an abrupt change of weighting scheme would result in a methodological discontinuity that would interfere with the historical analysis of stock price data. Prestbo continues:

> The Dow is unique in that it has been continuously calculated using the same methodology for more than a century. The importance of having this deep market history becomes abundantly clear when you start to analyze long-term market trends. . . Over The Dows 116-year history, there have been 23 business cycles. Wonder how the market will perform as we come out of a recession? Or how it might respond to changes in economic policy? The Dow is the index that offers the most complete dataset. Changing the methodology would essentially obliterate this history.

Attention in the financial press has therefore been focused on the need for a stock split, which would reduce the price of Apple shares to a level that could be accommodated by the questionable methodology of the Dow.

But an abrupt switch to a value-weighted index or the flawed artifice of a stock split are not the only available alternatives. We propose a modification to the Dow formula that largely preserves the historical integrity of the time series. At the same time, it allows for the inclusion of securities regardless of their market price and a smooth and gradual transition, as incumbent stocks are replaced, to a fully value-weighted index in the long run.

The proposed index is composed of two subindices, one price-weighted to respect the internal structure of the Dow, and the other value-weighted to apply to new entrants. There are two parameters, both of which are adjusted whenever a substitution is made. One of these maintains continuity in the value of the index, while the other ensures that the two subindices are weighted in proportion to their respective market capitalizations. Stock splits require a change in parameters (as in the case of the current Dow divisor) but only if the split occurs for a firm in the price-weighted subindex.

Once all incumbent firms have been replaced, the result will be a fully value-weighted index. In practice this could take several decades, as some incumbent firms are likely to be remain Dow components far into the future. But firms in the price-weighted component of the index that happen to have weights roughly commensurate with their market capitalization can be transferred with no loss of continuity to the value-weighted component. This procedure, which we call *bridging*, can accelerate the transition to a value-weighted index with minimal short term disruption.

We illustrate this set of principles with several examples.

2. Indexes

Suppose that there are n companies. Company i has share price p_i and outstanding shares s_i, so its market capitalization is given by $v_i \equiv p_i s_i$. An index based on these n companies is some function f defined on the vector $(p_1, s_1; \ldots; p_n, s_n)$.

f is a *price-weighted index* if

$$f(p_1, s_1; \ldots; p_n, s_n) = A \sum_{i=1}^{n} p_i$$

for some $A > 0$. It is a *value-weighted index* if

$$f(p_1, s_1; \ldots; p_n, s_n) = B \sum_{i=1}^{n} p_i s_i = B \sum_{i=1}^{n} v_i$$

for some $B > 0$.

It will help the exposition to think of f as being defined on a larger domain, say, for all subsets of m stocks, where $m \leq n$.

3. Substitution by Value-Weighting at the Margin

Let f be any index. The objective is to include a new stock, call it $n+1$ and drop an old stock, say stock 1. At the time of the change, we are in a particular environment; call it $(\bar{p}_1, \bar{s}_1; \ldots; \bar{p}_n, \bar{s}_n; \bar{p}_{n+1}, \bar{s}_{n+1})$. Proceed as follows. First, recalling that f is defined on subsets of stocks, define $\alpha > 0$ such that

$$\alpha f(\bar{p}_2, \bar{s}_2; \ldots; \bar{p}_n, \bar{s}_n) = \sum_{i=2}^{n} \bar{v}_i. \tag{1}$$

Now create the new index $g(p_2, s_2; \ldots; p_n, s_n; p_{n+1}, s_{n+1})$ by defining

$$g(p_2, s_2; \ldots; p_n, s_n; p_{n+1}, s_{n+1}) \equiv \delta \left[\alpha f(p_2, s_2; \ldots; p_n, s_n) + v_{n+1}\right], \tag{2}$$

where α is defined by (1), and δ is chosen so that

$$\delta \left[\alpha f(\bar{p}_2, \bar{s}_2; \ldots; \bar{p}_n, \bar{s}_n) + \bar{v}_{n+1}\right] = f(\bar{p}_1, \bar{s}_1; \ldots; \bar{p}_n, \bar{s}_n). \tag{3}$$

This ensures continuity of the index at the point of substitution.

The procedure defined above takes care of a single addition to an arbitrary current index, while value-weighting at the margin. How about subsequent additions? Suppose that we wish to add a second security $n+2$ at some later time, when the environment is $(\tilde{p}_2, \tilde{s}_2; \ldots; \tilde{p}_{n+2}, \tilde{s}_{n+2})$. Again recalling that f is defined on a subset of $n-2$ stocks, define $\alpha' > 0$ such that

$$\alpha' f(\tilde{p}_3, \tilde{s}_3; \ldots; \tilde{p}_n, \tilde{s}_n) = \sum_{i=3}^{n} \tilde{v}_i. \tag{4}$$

Now define a new index $g'(p_3, s_3; \ldots; p_{n+2}, s_{n+2})$ as

$$g'(p_3, s_3; \ldots; p_{n+2}, s_{n+2}) \equiv \delta' \left[\alpha' f(p_3, s_3; \ldots; p_n, s_n) + v_{n+1} + v_{n+2}\right], \tag{5}$$

where δ' is chosen so that

$$\delta' \left[\alpha' f(\tilde{p}_3, \tilde{s}_3; \ldots; \tilde{p}_n, \tilde{s}_n) + \tilde{v}_{n+1} + \tilde{v}_{n+2}\right] = g(\tilde{p}_2, \tilde{s}_2; \ldots; \tilde{p}_{n+1}, \tilde{s}_{n+1}). \tag{6}$$

As before, we have continuity of the index at the time of substitution. Clearly the same principle can be applied to add more stocks to replace the original incumbents as time unfolds. Once all incumbents have been replaced, we have a value-weighted index.

Remarks.

1. Note that the stocks in the index at any point in time can be partitioned into two groups: those that enter a subindex defined by f and those that enter a value-weighted subindex. All new additions that replace any asset in the former group can be made in accordance with the procedure identified above. But at some point in time it will be necessary to replace an asset in the latter group. This is easily accomplished. Suppose, for instance, that stock $n+2$ is to be replaced in the index (5) by a new stock, say $n+3$. Then we replace v_{n+2} by v_{n+3} in the formula and adjust the parameters δ and α in order to satisfy two conditions: continuity of the index value at the time of substitution, and proportionality of each subindex with their respective aggregate market capitalizations.

2. If a stock in the value-weighted subindex splits, it has no impact on the index and no adjustment need be made. If a stock in the subindex defined by f splits then an adjustment may be necessary, but its precise form will depend on the nature of f. In the next section we consider splits when f defines a price weighted index.

3. Our procedure yields exactly the same answer even if we arbitrarily change f by a multiplicative constant on any subset of companies. To see this, look at (1). Imagine that on the subset $2, \ldots, n-1$, we replace f by $\hat{f}$, where

$$\hat{f}(p_2, s_2; \ldots; p_n, s_n) = k f(p_2, s_2; \ldots; p_n, s_n)$$

for any $k > 0$. Then the corresponding value of α, call it $\hat{\alpha}$, is obviously related to α by $\hat{\alpha} = \alpha/k$ and so the subsequent equations (2) and (3) are entirely unaltered.

Exactly the same comments apply to the construction described in (4)–(6).

4. Application: Adding Apple to The Dow

The Dow is given by

$$f(p_1, s_1; \ldots; p_n, s_n) = \frac{1}{D} \sum_{i=1}^{n} p_i,$$

where D is the Dow divisor prior to substitution. The obvious extension of f to lower-dimensional subsets $\{m, \dots, n\}$, after dropping the first $m-1$ stocks, is

$$f(p_m, s_m; \dots; p_n, s_n) = A_m \sum_{i=m}^{n} p_i,$$

where A_m is any positive constant (by Remark 3 in Section 3, the precise choice of A_m is unimportant). Using this in (1), we see that for any particular vector $(\bar{p}_2, \bar{s}_2; \dots; \bar{p}_n, \bar{s}_n)$,

$$\alpha = \frac{1}{A_2} \frac{\sum_{i=2}^{n} \bar{p}_i \bar{s}_i}{\sum_{i=2}^{n} \bar{p}_i},$$

where we are agreeing to drop stock 1, say Alcoa. So the new index after adding Apple (our stock $n+1$) is given by

$$\delta \left[\frac{\sum_{i=2}^{n} \bar{p}_i \bar{s}_i}{\sum_{i=2}^{n} \bar{p}_i} \sum_{i=2}^{n} p_i + p_{n+1} s_{n+1} \right], \tag{7}$$

where δ is chosen to satisfy (3), so that there is no jump in the current value of the Dow.

More generally, once a total of $m-1$ incumbents have been replaced, the index takes the following two-parameter form:

$$\delta \left[\alpha \sum_{i=m}^{n} p_i + \sum_{i=n+1}^{n+m} v_i \right].$$

The two parameters δ and α are adjusted each time a substitution is made: δ to maintain continuity in the value of the index, and α to ensure that each of the two subcomponents receive weights in proportion to their market capitalizations. Stock splits require changes in both parameters but only if they occur for firms in the price-weighted subcomponent.

This construction allows us to add apples and oranges. At the times of substitution, it is tantamount to adding valuations just like a value-weighted index would, but in subsequent movements of prices it respects the internal structure of The Dow (projected onto the first n companies).

Of course the new index is neither price-weighted nor value-weighted. But our iterative procedure allows us to keep adding new stocks. It is obvious that once the original n stocks have dropped out, the procedure generates a fully value-weighted index.

5. An Implementation

Table 1 shows the 30 current Dow components in decreasing order of market capitalization as of August 3, 2012. The index itself stood at 13096.17 at this time, and the Dow divisor was approximately $D = 0.1321$. The sixth column in the table contains the weights of the respective stocks in the computation of the Dow, which are simply proportional to prices. Hence IBM, for example, has a weight in the Dow that is more than

Symbol	Name	Price	Shares	Capitalization	P-Weight	V-Weight
AA	Alcoa	8.37	1,066,890,000	8,929,869,300	0.48%	0.22%
TRV	Travelers	63.71	385,358,000	24,551,158,180	3.68%	0.62%
HPQ	Hewlett-Packard	18.26	1,971,830,000	36,005,615,800	1.06%	0.91%
DD	E.I. du Pont	49.47	930,373,000	46,025,552,310	2.86%	1.16%
BA	Boeing	72.81	751,798,000	54,738,412,380	4.21%	1.38%
CAT	Caterpillar	85.02	652,500,000	55,475,550,000	4.91%	1.39%
MMM	3M	91.7	691,320,000	63,394,044,000	5.30%	1.59%
AXP	American Express	57.61	1,133,650,000	65,309,576,500	3.33%	1.64%
UTX	United	76.81	911,787,000	70,034,359,470	4.44%	1.76%
KFT	Kraft Foods	40.51	1,772,988,000	71,823,743,880	2.34%	1.81%
HD	Home Depot	52.29	1,530,660,000	80,038,211,400	3.02%	2.01%
BAC	Bank of America	7.43	10,777,000,000	80,073,110,000	0.43%	2.01%
CSCO	Cisco	16.35	5,356,880,000	87,584,988,000	0.94%	2.20%
DIS	Walt Disney	49.77	1,787,360,000	88,956,907,200	2.88%	2.24%
MCD	McDonald's	89.59	1,016,240,000	91,044,941,600	5.18%	2.29%
VZ	Verizon	44.46	2,848,880,000	126,661,204,800	2.57%	3.18%
INTC	Intel	26.23	5,003,000,000	131,228,690,000	1.52%	3.30%
MRK	Merck	44.15	3,041,560,000	134,284,874,000	2.55%	3.38%
JPM	JP Morgan Chase	36.09	3,806,670,000	137,382,720,300	2.09%	3.45%
PG	Procter & Gamble	65.5	2,740,110,000	179,477,205,000	3.79%	4.51%
PFE	Pfizer	24.28	7,488,140,000	181,812,039,200	1.40%	4.57%
KO	Coca-Cola	80.83	2,250,960,000	181,945,096,800	4.67%	4.57%
JNJ	Johnson & Johnson	69.12	2,757,040,000	190,566,604,800	3.99%	4.79%
CVX	Chevron	111.12	1,962,140,000	218,032,996,800	6.42%	5.48%
T	AT&T	37.58	5,805,000,000	218,151,900,000	2.17%	5.48%
GE	General Electric	20.96	10,558,800,000	221,312,448,000	1.21%	5.56%
IBM	IBM	198.52	1,142,780,000	226,864,685,600	11.47%	5.70%
MSFT	Microsoft	29.75	8,383,400,000	249,406,150,000	1.72%	6.27%
WMT	Wal-Mart	74.55	3,383,540,000	252,242,907,000	4.31%	6.34%
XOM	Exxon Mobil	87.55	4,615,940,000	404,125,547,000	5.06%	10.16%

Table 1. Dow Components as of August 3, 2012

nine times that of GE and almost seven times that of Microsoft, even though all three firms are of comparable market capitalization. Meanwhile AA and BAC have negligible weights and hence little impact on day-to-day movements of the index.

Now consider the possibility of replacing Alcoa with Apple, which had a price of 615.70 and market capitalization of 577 billion at this time. If this substitution were made without altering the method by which the Dow is computed, then the new index would be given by

$$\frac{1}{D'} \sum_{i=2}^{31} p_i,$$

where the modified Dow divisor would be approximately 0.1785. Apple would carry a weight of more than 26% in the new index, and both the volatility and the day-to-day changes in value of the index would be heavily dependent on the behavior of this one security. And if Google, with a share price of 641.33, were also included (in place of Travelers for instance) we would have an even more absurd scenario with the two new entrants accounting for more than 43% of the Dow. This is precisely why the inclusion of these firms has been resisted to date.

Yet Apple clearly belongs in any representative stock market index. So does Google, which would place comfortably in the top ten components by capitalization were it to be included. This could be done without seriously compromising the historical integrity of the time series by adopting the approach described here. If Apple were to replace Alcoa under our proposed procedure, the new index would be

$$\delta \left[\beta \sum_{i=2}^{30} p_i + v_{31} \right]$$

where

$$\beta = \frac{\sum_{i=2}^{30} \bar{v}_i}{\sum_{i=2}^{30} \bar{p}_i} \approx 2.30 \times 10^9$$

and $\delta = 2.88 \times 10^{-9}$ to ensure continuity of the index at the time of substitution. The initial weight of Apple in the new index will simply equal its market capitalization relative to total market capitalization of the components, which amounts to less than 13%. This will change over time with changes in prices, and will also change as additional substitutions are made. But note that the initial weight of Apple in our proposed modification to the Dow would be just slightly greater than the weight of IBM in the current index, even though IBM has half the market capitalization of Apple. If we can tolerate IBM on these terms, why not Apple?

6. Bridging

Reflect again on just how the Dow is computed. In the Introduction, we used the example of Chevron and AT&T. We showed that Chevron is three times as important in affecting the Dow as AT&T is. The penultimate column of Table 1 shows the implicit "price weights" given to the Dow 30 under the current method of computing the index. Chevron's price weight is 6.42%, as opposed to AT&T's 2.17%.

Indeed, the Dow is rife with such examples. Compare the price weights in the penultimate column of Table 1 with the market value weights in the very last column. Note how

Chevron and AT&T have identical (or near-identical) market value weights, as do Intel and Merck, or Coca Cola and Pfizer, or Microsoft and Walmart. Yet the members in these pairs of companies are accorded very different roles in the Dow.

As we've discussed, there is little we can do by way of radical change with such "incumbent companies" for fear of destroying the historical continuity of the index. But for companies that have the coincidental property that their price weight happens to equal their market value weight (or is within some tolerance threshold of the latter), we might consider migrating them over, or "bridging" them, to a value-weighted system. This would speed up the transition to eventual value-weighting.

More formally, fix a particular environment $(\bar{p}_1, \bar{s}_1; \ldots; \bar{p}_n, \bar{s}_n)$. Value weight equals price weight for a company k if

$$\frac{\bar{p}_k}{\sum_i \bar{p}_i} = \frac{\bar{v}_k}{\sum_i \bar{v}_i}. \tag{8}$$

In this case, we can remove stock k from the price-weighted component and bring it back in again using the value-weighted procedure without affecting the total value of the index or the importance of that stock relative to any of the other companies in the Dow. To see this, suppose without loss of generality that $k = 1$. Then following the procedure leading up to equation (7), we have the new index

$$\delta_1 \left[\frac{\sum_{i=2}^n \bar{p}_i \bar{s}_i}{\sum_{i=2}^n \bar{p}_i} \sum_{i=2}^n p_i + p_1 s_1 \right] \tag{9}$$

where δ_1 is chosen to satisfy (3) as before. This is exactly the same procedure except that we removed and added the same company (company 1 in this case). Of course the value of the Dow is unaltered, just as before, by the choice of δ_1. But more importantly, we've left entirely untouched the relative importance of company 1 to every other company in the entire index, "locally" evaluated at the existing environment. Under price weighting, the relative impact of a one-percent change in stock 1 relative to any other stock j was equal to p_1/p_j. After the change — given (9) — that relative impact is

$$p_1 s_1 / p_j \frac{\sum_{i=2}^n \bar{p}_i \bar{s}_i}{\sum_{i=2}^n \bar{p}_i}.$$

Generally speaking, this ratio will diverge from p_i/p_j as the environment continues to evolve, but the point is that "locally" around the original environment $(\bar{p}_1, \bar{s}_1; \ldots; \bar{p}_n, \bar{s}_n)$ when the change took place, the ratio is no different:

$$\bar{p}_1 \bar{s}_1 / \bar{p}_j \frac{\sum_{i=2}^n \bar{p}_i \bar{s}_i}{\sum_{i=2}^n \bar{p}_i} = \bar{p}_1 / \bar{p}_j.$$

To see this, use equation (8).[2]

[2] Equation (8) readily implies that $\bar{p}_1 / \sum_{i=2}^n \bar{p}_i = \bar{v}_1 / \sum_{i=2}^n \bar{v}_i = \bar{p}_1 \bar{s}_1 / \sum_{i=2}^n \bar{p}_i \bar{s}_i$, and now the desired equality follows right away.

The bridging process speeds up convergence to a value-weighted index without compromising the historical continuity of the Dow. To be sure, it can only be applied to incumbent companies for which the fortuitous equality described in (8) happens to hold. Is that true today for any of the stocks in the Dow? The answer appears to be in the affirmative for Coca Cola (price weight 4.67% versus value weight 4.57%), and perhaps Disney (2.88% versus 2.24%). We recommend bridging them.