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Poverty and Disequalization

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Abstract

We study the intergenerational transmission of inequality using a model in which parents can make both financial and occupational bequests to their children. An equal steady state with high per capita skill can co-exist with unequal steady states with low per capita skill. We investigate dynamics starting from arbitrary initial conditions. The main result is that even if a country starts with a perfectly equal wealth distribution, it converges to an unequal steady state if its initial per capita wealth falls below a threshold, and to the equal steady state otherwise. Hence initial poverty (even with perfect equality) can generate long-term inequality, and undermine economic development.

KEYWORDS: inequality, poverty traps, convergence, human capital

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1 Introduction

Whether a market economy is inherently equalizing or disequalizing in the presence of parental bequests and credit market imperfections is a question that has received much attention from macro-development theorists over the past two decades. Variants of the Solow model, such as Becker and Tomes (1979, 1986) and Loury (1981)) predict that the market is fundamentally *equalizing*: in the absence of ongoing "shocks", wealth differences between households tend to vanish and disappear in the long run. These models are based on the assumption of a continuum of investment options with an exogenously given, concave pattern of returns. On the other hand, theories of discrete occupational choice with credit constraints (e.g., Banerjee and Newman (1993), Galor and Zeira (1993), Ghatak and Jiang (2002)) are compatible with a multiplicity of steady states.[1] In a broad sense the latter set of models suggest markets are *neutral* with regard to inequality: societies that start equal remain equal in the long run, while those that start substantially unequal converge to unequal steady states (associated with lower levels of per capita income and human capital). Variations in the level of development across countries can thus be explained by historical differences in inequality.

A related literature (Ray (1990, 2006), Ljungqvist (1993), Freeman (1996), Mookherjee and Ray (2003)) takes a stronger stance, predicting that inequality must endogenously arise, though the extent of it may depend on initial inequality. In particular, even if an economy starts perfectly equal, it must end up unequal in the long run. Hence this literature is associated with the view that markets are fundamentally "disequalizing". These models make the key assumption that investment returns are endogenously determined by economy-wide investment patterns, and occupations with disparate entry costs are essential in production.[2] This ensures that at any date the market equilibrium will always be characterized by substantial occupational diversity. Even if all households start equal, occupations with differing entry costs must be chosen by their young, which results in inequality in the following generation. And once this inequality appears, it tends to be reinforced in successive generations. In contrast, both the convergence and neutrality models permit all agents in the economy to be concentrated in a single occupation (or in disparate occupations with identical net earnings).

In an important sense, the endogenous inequality models overstate the argument for the disequalizing role of markets, because they do not incorporate financial bequests. If the theory is extended to allow parents to leave financial bequests as

[1] This would also be true of competitive versions of the Solow model in which the rates of return to individual households are unchanging in investment.

[2] The neutrality models do not require that returns to investment be endogenously determined (as in Galor-Zeira (1993)), and no occupational category is necessarily presumed to be essential.

well as invest in their children's human capital, inequality is no longer inevitable in the long run. Those that do not invest in their children's human capital could leave them with sufficient financial bequests to ensure that they end up with identical lifetime incomes and consumption. Hence equal steady states may exist in which investments in the two kinds of capital are negatively correlated. This is shown in an earlier paper of ours (Mookherjee and Ray (2009)) which focused on a characterization of steady states. It was shown there that while a continuum of unequal steady states always exists, an equal steady state also exists, provided the "span" of entry costs across disparate essential occupations is not too large relative to the strength of parental altruism.

This leaves open the question of transition dynamics: if both equal and unequal steady states coexist, which of these will the economy converge to from a starting position of perfect equality? To what extent is inequality truly a "necessary" outcome?

Our main result is that starting from perfect equality, *markets are disequalizing* (i.e., the economy converges to an unequal steady state) *if and and only if the economy starts out sufficiently poor* (i.e., starting per capita wealth is sufficiently low). Unequal steady states involve lower per capita income and skill than the single equal steady state in the model, thus corresponding to a lower long-run level of development. Hence the model generates a novel connection between the initial level of poverty (rather than inequality) and the long-run level of development. Whether the market is equalizing or disequalizing thus depends on how well-off the economy is to start with.

The analysis can be extended to accommodate initial inequality as well. We can obtain a detailed characterization of the complete dynamic of the market equilibrium, when the economy starts from an arbitrary non-degenerate wealth distribution. It turns out that while the economy always converges to a steady state, the presence of initial inequality raises the threshold of initial per capita wealth that the economy has to surpass in order to converge to an equal steady state. Hence both historical poverty and inequality matter for the subsequent market dynamic.

This result contrasts with the neutrality models, which predict that equality once achieved will tend to persist forever (in the absence of any random shocks to ability or income luck).

Our argument is based on an overlapping generations model with a bequest motive, a single consumption good, and two occupations (skilled and unskilled). The skilled occupation requires an exogenous training cost. Production depends on both skilled and unskilled labor as well as physical capital; both labor types are assumed to be needed in production. There is no uncertainty in the model, and the credit market imperfection is represented by a missing credit market and the assumption that parents must leave non-negative financial bequests to their children.

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To keep the analysis tractable we assume the return on physical capital (or financial wealth) is fixed, owing to the presence of an international financial market.

We impose the "limited persistence" condition of Becker and Tomes (1979), which ensures that the marginal propensity of a parent to bequeath wealth to his child lies between 0 and 1. As in Becker-Tomes, this ensures that bequests would be fundamentally equalizing in the absence of occupational choice. On the other hand, the rate of return on human capital is endogenous: skilled and unskilled wages depend on the aggregate supplies of skilled and unskilled labor. In general, and for the reasons emphasized in the endogenous inequality models, investments in human capital will tend to be disequalizing. The net effect — equalization or disequalization — will then depend on the relative importance of physical and human capital, which is also endogenously determined.

The results relating initial poverty to long-run outcomes capture the intuitive idea that markets do not work well in the presence of poverty. Credit market imperfections bite more strongly to inhibit productive investments. Yet some of these investments are essential, and in equilibrium some agents are provided the incentive to undertake these despite their poverty. Such incentives necessitate a large skill premium, i.e., a high rate of return on skilled occupations *vis-a-vis* unskilled occupations. Hence sufficiently high initial levels of (equal) poverty across households imply high levels of inequality in earnings across households choosing disparate occupations in the following generation. The high sacrifice of the very first generation of parents that invest in their children's skill is compensated by the substantially higher earnings of their children. At the same time, the earnings of the unskilled in the second generation are low. Consequently, second-generation unskilled parents do not want to invest in their children's education. The occupational distribution does not change thereafter: a vicious circle has then set in, and the economy must converge to an unequal steady state.

In contrast, if the economy starts with a sufficiently high level of per capita wealth, parents are wealthy enough to invest in human capital at a rate which ensures that the rate of return on human capital is the same as that on physical capital. Each parent then faces a concave investment technology. Since they all have equal wealth in the first generation, so will their children, and the situation reproduces itself. The economy remains equal forever, at the equal steady state from the first generation itself.

Finally, if the economy starts with an intermediate level of per capita wealth, some inequality in human capital earnings appears in the succeeding generation. But this is overwhelmed in due course by the equalizing effects of financial bequests: unskilled households pull themselves up by their bootstraps. This in turn lowers the skill premium and helps other unskilled households to raise their earnings, and thereby to educate their children. A virtuous cycle is then instituted, whereby inequality falls over time, and the economy converges eventually to an equal steady state. The evolution of inequality and per capita income in this intermediate case resembles a Kuznets curve.

2 Model

Consider a standard OLG economy, with a continuum of households and a sequence of generations $t = 0,1,2,\ldots$. A household in any given generation is represented by a single *parent* and a single *child*, the latter being the parent of this household in the following generation. There is a single consumption good, and two occupations, one skilled (s) which requires a fixed training cost of x (denominated in units of the consumption good), and the other unskilled (u) which requires no training.

The consumption good is produced from physical capital, unskilled and skilled labor, using a smooth, strictly quasiconcave CRS production function, satisfying the Inada endpoint conditons in labor inputs. The rate of return on physical capital, which is also the rate of return on financial bequests, is exogenously fixed at r.

Given the interest rate, the "reduced" production function — after netting out optimally used physical capital — can be expressed by a smooth CRS function $f(\lambda)$ of the proportion λ of skilled households in the economy. (The reduced function f depends on r, but we suppress this dependence in the notation to avoid clutter.) Denote the marginal products of the two occupations by $f_s(\lambda)$ and $f_u(\lambda)$. These are respectively decreasing and increasing functions, and the Inada conditions guarantee that $f_s(\lambda) \to \infty(0)$ and $f_u(\lambda) \to 0(\infty)$ as $\lambda \to 0(\infty)$. In particular, there exists $\tilde{\lambda} \in (0,1)$ such that $f_s(\lambda) > f_u(\lambda)$ if and only if $\lambda < \tilde{\lambda}$.

Skilled agents can work as unskilled labor if they choose to. If the skill ratio in any generation exceeds $\tilde{\lambda}$, the equilibrium of the labor market will generate equal wages for both occupations, and the common wage will be $f_s(\tilde{\lambda}) = f_u(\tilde{\lambda}) \equiv \tilde{w}$. Even though we haven't defined an equilibrium yet, we know this outcome cannot arise, as the sacrifice of parents of skilled children would have been in vain. Therefore the relevant portion of the state space is $(0,\tilde{\lambda})$, with wages equal to marginal products in the two occupations. Let $w_s(\lambda) = f_s(\lambda)$ and $w_u(\lambda) = f_u(\lambda)$ be the resulting wages of the skilled and unskilled occupations.

An adult in any given generation has wealth W from two sources: a market-determined wage that depends on her occupation, and financial assets that represent the (interest-updated) result of financial bequests received from her parent. In turn, she allocates W between current consumption c, a non-negative financial bequest b to her child, and training costs $x(h)$ for her child's future occupation h (where $x(s) = x, x(u) = 0$).

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The resulting wealth of the child in the next generation will be $W' \equiv (1 + r)b + w_h(\lambda')$, where λ' is next generation's skill ratio. A parent derives utility $U(c) + V(W')$, where U and V are increasing, smooth, and strictly concave functions. This is essentially a "paternalistic" bequest motive.[3]

The specification of preferences implies that a parent and her child will have identical preferences over the choice of occupation for the latter, given the aggregate outlay invested by the parent on behalf of the child. In other words, an equivalent formulation is the following. Given her own lifetime wealth W, a parent can decide on how much to consume c ($\leq W$), with the remainder $B \equiv W - c$ left as a financial bequest. The child optimally decides how to allocate this bequest between financial wealth and skill acquisition, but in the latter case $B - x$ must be nonnegative. Let $\mu(B;\lambda')$ denote the maximized wealth of an adult who receives a total bequest of B from her parent, and who lives in a generation with skill ratio λ'. Then we can formulate the parent's problem as selection of B (given W) to maximize $U(W - B) + V(\mu(B;\lambda'))$ subject to $B \leq W$, where the skill ratio anticipated for the following generation is λ'.

We can now define a competitive equilibrium sequence (with perfect foresight) resulting from an arbitrary initial wealth distribution. At $t = 0$, each dynasty or household i starts with a given bequest $B_i(0)$; the distribution over these bequests across households is given. In each subsequent generation t this household i will inherit a bequest $B_i(t)$ chosen by her parent. The equilibrium specifies a choice of occupation and consumption for each household in each generation, and a corresponding sequence of skill ratios λ_t that this gives rise to upon aggregating across education decisions of different households.

Specifically, for any $t \geq 0$, household i will select an occupation to maximize her wealth. She must remain unskilled if $B_i(t) < x$. Otherwise, she selects the unskilled occupation if $(1+r)B_i(t) + w_u(\lambda_t) > (1+r)(B_i(t) - x) + w_s(\lambda_t)$, the skilled occupation if this inequality is reversed, and either of the two occupations if it is an equality. Household i at date t ends up with wealth $W_i(t) \equiv \mu(B_i(t);\lambda_t)$ which equals $(1+r)B_i(t) + w_u(\lambda_t)$ if $B_i(t) < x$, and the maximum of $(1+r)B_i(t) + w_u(\lambda_t)$ and $(1+r)(B_i(t) - x) + w_s(\lambda_t)$ otherwise. She then selects a bequest $B = B_i(t+1)$ to maximize

$$U(W_i(t) - B) + V(\mu(B;\lambda_{t+1})) \tag{1}$$

subject to $B \leq W_i(t)$. Finally, λ_t equals the measure of households in generation t who select the skilled occupation.

[3] In contrast to conventional warm-glow formulations, parents here care not just about the size of the bequest but rather the resulting wealth of their children.

Note that we have constrained consumption to be non-negative, which in turn restricts bequests to not exceed lifetime wealths. This implies that equilibria are not well-defined in any generation where no household earns a wealth of at least x. In such a situation, no child can inherit enough to be able to afford an education, which implies there will be no skilled agent in the succeeding generation, and the consumption good cannot be produced. There are various ways of modifying the model to address this problem, e.g., allowing agents the option of borrowing at 'high' costs, whereby scarcity of skilled agents will raise the skill premium sufficiently to elicit the required supply. We prefer to avoid this problem by restricting attention to initial wealth distributions $F_0(W)$ with the property that $F_0(x) < 1$, i.e., a positive measure of households inherit wealth exceeding x. This is easily checked to ensure that the same property will hold in every succeeding generation, i.e., $F_t(x) < 1$ for all t. This is because in any generation where there is a positive fraction of agents who can afford to acquire education, a positive fraction will indeed acquire education (owing to the Inada conditions), and thereafter earn a lifetime wealth of at least x. Moreover a positive fraction amongst them will bequeath at least x to their children, owing to the nature of the paternalistic altruism which is sensitive to the wealth acquired by children.[4]

2.1 Steady States

An equilibrium is a *steady state* if the joint distribution over bequests and occupations does not change across generations. In principle, a steady state could be associated with wealth changes within dynasties, but standard "single-crossing" arguments rule this out. Since parents must bear the sacrifice of investing in their children (owing to their inability to borrow from them), wealthier parents will leave more bequests. This is evident from expression (1) for the objective function of parents when they choose their bequests: the strict concavity of U, combined with strict monotonicity of V and of μ in B implies that a higher value of $W_i(t)$ is associated with a weakly higher bequest incentive. Hence every steady state is associated with zero wealth mobility.

An *equal steady state* is one in which all households have the same wealth in each generation.

Equal steady states do not always exist. We follow Mookherjee and Ray (2009) here in providing a condition for existence of an equal steady state.

Let $\omega(W, w_0, r)$ denote the wealth of a child whose parent's wealth is W, in a hypothetical "finance-only" world *a la* Becker-Tomes (1979) in which there are

[4] For if this were not true, the skilled wage in the next generation would be infinitely large, which would motivate parents with a lifetime wealth exceeding x to bequeath at least x.

only financial assets and everyone earns an exogenous wage of w_0, so that a bequest of B generates offspring wealth of $w_0 + (1+r)B$. Hence $\omega(W; w_0, r) = w_0 + (1 + r)B$, where B maximizes $U(W - B) + V(w_0 + (1+r)B)$ subject to $B \in [0, W]$.

The Becker-Tomes assumption of *Limited Persistence (LP)* states that the 'marginal propensity to transmit wealth', defined as the slope of ω with respect to W lies between 0 and 1. We shall assume this property holds for the remainder of this paper.

Next, define $\Omega(w_0, r)$ as the solution for W in the equation $\omega(W; w_0, r) = W$. This can be interpreted as "long-run" wealth, and is well-defined given the limited persistence assumption. Finally, let λ^* denote the skill ratio at which the rate of return to skill acquisition equals r, i.e., $w_s(\lambda^*) - w_u(\lambda^*) = (1+r)x$.

The condition that ensures existence of an equal steady state is

$$\Omega(w_u(\lambda^*), r) \geq w_s(\lambda^*). \tag{2}$$

PROPOSITION **1.** *An equal steady state must involve $\lambda = \lambda^*$. It exists if and only if (2) holds.*

Condition (2) can be interpreted as requiring the training cost x to not be too large, relative to the extent of parental altruism.[5] To illustrate using an example with constant-elasticity preferences, assume V equals δU for some discount factor δ and $u(c) = c^{1-\sigma}/(1-\sigma)$ for some $\sigma > 0$. Then $\omega(W; w, r) = \frac{(1+r)\rho}{1+\rho+r} W + \frac{\rho}{1+\rho+r} w$ if $\rho W \geq w$, and equals zero otherwise, where $\rho \equiv [\delta(1+r)]^{1/\sigma}$. In this example, the Becker-Tomes limited persistence assumption reduces to the condition

$$\rho \equiv [\delta(1+r)]^{1/\sigma} < 1 + \frac{1}{r}. \tag{3}$$

Under this assumption, there is a unique limit wealth $\Omega(w; r)$ which takes the following form: $\Omega(w, r) = w$ if $\rho \leq 1$, and equals $\frac{\rho}{1-r(\rho-1)} w$ otherwise. Condition (2) then requires that $\rho \geq \rho^*$, where the latter is defined by the solution to

$$\frac{\rho^*}{1 - r(\rho^* - 1)} = \frac{w_s(\lambda^*)}{w_u(\lambda^*)} \equiv 1 + (1+r)\frac{x}{w_u(\lambda^*)}. \tag{4}$$

Since ρ^* is increasing in x, it follows that the condition requires training cost x to be low relative to the extent of parental altruism ρ.

[5] This is because a rise in training cost x is associated with a lower λ^*, a higher skilled wage $w_s(\lambda^*)$ and a lower unskilled wage $w_u(\lambda^*)$, which causes the left-hand-side of (2) to fall and the right-hand-side to rise. Also note that at $x = 0$ condition (2) must be satisfied, since at $x = 0$, we must have $\lambda^* = \bar{\lambda}$ where skilled and unskilled wages are equalized at $\bar{w}$. Then the left-hand-side of (2) is by definition at least as large as the unskilled wage $\bar{w}$, while the right-hand-side equals the skilled wage $\bar{w}$.

The following argument establishes necessity of (2) for the existence of an equal steady state. In an equal steady state, all households end up with equal wealth at every date. And some of them must choose to be skilled, others unskilled. So they must be indifferent between investing and not investing in skill. Moreover, all children must end up with the same wealth. This requires the rate of return to skill be equal to r (if it were higher, those skilled would end up with higher wealth). Hence the skill ratio must be λ^*. Then the return to investment is linear, just as in a finance-only world with base wage $w_u(\lambda^*)$ and interest rate r. This implies a common steady state wealth level of $\Omega(w_u(\lambda^*), r))$, which must be at least as high as the skilled wage (since those skilled earn at least this much, and in addition may inherit some financial bequest as well). Hence condition (2) must hold. It turns out this condition is also sufficient for an equal steady state to exist: see Mookherjee and Ray (2009) for details.

At the same time, a continuum of unequal steady states also exists. For instance, consider any λ such that $\Omega(w_u(\lambda), r) \leq x$. Then it is evident that $\lambda < \lambda^*$, since $w_s(\lambda^*)$ must exceed x. It follows that λ is the skill ratio of an unequal steady state. This is because the unskilled wage is low enough that the corresponding finance-only steady state wealth falls below x. In that case the bequest must also fall below x, and their children cannot afford education. On the other hand, skilled families will want to invest in education at a steady skill ratio of λ, since they are willing to do so even at the equal steady state skill ratio λ^*, and their incentive (and ability to afford) to invest in education is larger when the skill ratio is lower.

Unequal steady states can be characterized as follows. Use $Z(W; w_0)$ to denote the indirect utility of a parent in a finance-only world with current wealth W and flow earnings w_0 of the child (at the given interest rate, which we are suppressing in the notation). Also from now on, we suppress r in the notation for the finance-only steady state wealth, and denote it by $\Omega(w_0)$.

PROPOSITION **2.** *Assume (2). Then λ is an unequal steady state skill ratio if and only if*

$$\lambda < \lambda^* \tag{5}$$

and in addition

$$\max_{b \geq 0} [U(\Omega(w_u) - x - b) + V(w_s + b(1+r))] \leq Z(\Omega(w_u), w_u) \tag{6}$$

where $w_u = w_u(\lambda), w_s = w_s(\lambda)$.

The argument is straightforward: if $\lambda = \lambda^*$ then we are effectively in a finance-only world with a linear investment frontier and a constant rate of return r on all investments, where there cannot be any long run wealth inequality. So

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$\lambda < \lambda^*$ is necessary for wealth inequality. Unequal steady states must involve a nonconvexity in investment returns: up to an investment of x only financial bequests are possible, while at x there is a discontinuous upward jump in investment returns (the size of which depends on the gap between the rate of return on human and financial capital). Since the return on human capital is higher, any agent wishing to invest x or more must first invest in education, and keep the remainder in financial assets.

If $\lambda < \lambda^*$ then skilled families have wealth $\Omega(w_s) > \Omega(w_s^*)$. They will want to invest in their children's education because they wanted to invest at least x when λ equalled λ^*, by (2). Now they are even richer and the returns to investing in education are even higher.

Hence one only needs to check the incentives of the unskilled: they must not want to invest x or more. This is the role of condition (6). In steady state unskilled households must have a wealth of $\Omega(w_u)$. The left side is the maximum payoff of a parent with this wealth conditional on investing a total of at least x in her child. The right side is what she attains with financial investments alone. This condition implies, in particular, that the wealth of the unskilled falls below the skilled wage: $\Omega(w_u) < w_s$.[6] So there must be wealth inequality in these steady states.

Let Γ denote the set of unequal steady state skill ratios. In general Γ consists of a continuum of skill ratios, because it is characterized by a set of inequalities.[7] Hence a continuum of unequal steady states co-exists with the equal steady state, when (2) holds. Moreover, within the set of unequal steady states, those with a lower skill ratio are associated with lower per capita income and higher inequality.

In what follows, we shall assume that (2) holds, in order to examine the conditions for equalization or disequalization, i.e., convergence to either kind of steady state, from different initial conditions.

2.2 Dynamics

The following result serves as a prelude to the dynamic analysis.

PROPOSITION **3.** *Assume (2), so that both the equal steady state with skill ratio λ^* and a continuum of unequal steady states Γ exist. Then:*
(a) *From an arbitrary initial wealth distribution at date* 0 *satisfying* $F_0(x) < 1$*, the economy converges to a steady state.*

[6] Otherwise unskilled parents would be investing at least x in their children, in which case they would be better off educating them rather than provide only financial bequests.

[7] If λ is an unequal steady state where the constraint (6) holds as a strict inequality, then an open neighborhood of it also satisfies (6) as a strict inequality.

(b) *If the equilibrium skill ratio in the first generation exceeds* $\bar{\lambda}$ *(the highest skill ratio across all unequal steady states), the economy converges to the equal steady state. In this case the skill ratio rises monotonically and converges to* λ^*.
(c) *If the equilibrium skill ratio in the first generation falls below* $\bar{\lambda}$*, the equilibrium converges to an unequal steady state. If the first generation skill ratio is an unequal steady state skill ratio, the equilibrium skill ratio sequence converges to that steady state, i.e., the equilibrium skill ratio is stationary. If the first generation skill ratio is not an unequal steady state skill ratio, then the equilibrium skill ratio rises across successive generations and converges to the smallest steady state skill ratio lying above the first generation skill ratio.*

This proposition says that the dynamics depend on the historical wealth distribution, which determines the equilibrium skill ratio in the first generation.

We now state our main result, pertaining to the case where all families start with equal initial wealth.

PROPOSITION **4.** *Suppose all families start with the same wealth* $W_0(> x)$*. Then there exists a threshold* $\bar{W}(> x)$ *such that if* $W_0 \leq \bar{W}$*, the economy converges to an unequal steady state, while if* $W_0 > \bar{W}$ *it converges to the equal steady state.*

This result distinguishes the model from the previous "neutral to inequality" literature, in which initial equality always implies equality for ever thereafter. It also distinguishes it from the "endogenous inequality" literature in which convergence (if it occurs) from any initial condition must be to an unequal steady state. We obtain a more nuanced theory which combines elements of both literatures. Societies that start perfectly equal converge to an unequal steady state if they are sufficiently poor initially; otherwise they converge to an equal steady state. Initial and eventual per capita wealth are then positively related, and poor countries do not eventually catch up with rich countries. The market serves to disequalize wealth in poor countries, and to equalize it non-poor countries.

Initial wealth matters because it affects the (initial) incentive to invest in human capital, given the presence of borrowing constraints. Sufficiently poor countries cannot make the required initial investments in skill to boost the unskilled wage to a level that can initiate a virtuous upward spiral for unskilled families. In the initial generation all families have the same wealth and investment preferences. To ensure the supply of some skilled people in the following generation, this "symmetry must be broken": all families must be indifferent to skill acquisition, and some families must choose to invest and others not. But this symmetry-breaking has irreversible consequences for subsequent inequality: skilled families have peristently higher wealth than unskilled families. Earnings inequality emerges and remains stationary. Over succeeding generations the wealth of the unskilled fall, while those of

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the skilled rise, so the operation of financial transfers exacerbates the perpetuation of inequality. In this region, the insights of the "endogenous inequality" literature prevail. Indeed, the evolution of inequality is reinforced by the existence of financial capital.

On the other hand, if an economy starts perfectly equal *and* sufficiently rich, then $\lambda = \lambda^*$ in the very first generation itself. Again, symmetry is broken, with some households investing in human capital and others not, while all are indifferent. But this time there is no subsequent inequality: those not investing in human capital invest in financial capital instead. In contrast to the economies which start poor, here $\lambda = \lambda^*$ in the first generation, ensuring that the rates of return on both kinds of capital are equalized. This implies that the composition of investments between the two forms of capital does not matter, and perfect wealth equalization must obtain in the next generation as well. The same logic repeats itself generation after generation, with financial transfers perfectly offsetting differences in educational investments throughout. This is exactly the logic of perpetuation of equality in the "neutral to inequality" literature: endogenous inequality never has the opportunity to manifest itself.

There is an intermediate third case, in which some elements of "equalization" and "disequalization" both appear, but the former prevail. If the economy starts rich but not "too rich", then the skill ratio in the first generation is less than λ^*, so that wealth inequality emerges initially. In this case, however, unskilled wages are still high enough so that sufficient financial bequests are made, causing the wealth of the unskilled to grow. In turn this causes the demand for education (and hence λ) to grow, raising unskilled wages even further, and lowering the wage gap between skilled and unskilled. The wealth of the unskilled rise faster than that of the skilled, resulting in convergence. Here financial bequests induce "trickle down" and the market is equalizing.

What about the more general case where the initial wealth distribution is non-degenerate? Then initial inequality also matters: even for a country with high initial per capita wealth. If this wealth is distributed sufficiently unequally the equilibrium skill ratio at the beginning can fall below $\bar{\lambda}$, causing the economy to converge to an unequal steady state. This, of course, is familiar from existing literature.

3 Concluding Comments

This paper contributes to a growing literature on history-dependence in wealth inequality. We study a model in which a variety of steady states are possible: some involving perfect equality and some involving inequality. This distinguishes our model from previous endogenous inequality literature, which either does not allow financial bequests or otherwise imposes conditions that rule out such coexistence. At the same time, our model is different from a literature that emphasizes "neutrality" to (in-)equality, where equality once established is maintained for ever.

In contrast, by emphasizing the endogeneity of skilled and unskilled wages to the overall supplies of skilled and unskilled labor, we argue that initial poverty undermines the stability of equality. We show this by studying initial conditions that all exhibit perfect equality but vary in the *level* of initial wealth. Both skilled and unskilled labor are needed in production, so in the theory we propose a perfectly equal initial condition must exhibit symmetry-breaking, with some families choosing skills and others remaining unskilled. When initial wealth is very low, unskilled families perpetually lag behind and inequality grows over generations, even from a starting point of perfect equality. When initial wealth is very high, symmetry breaking can be achieved at no cost to the unskilled, who can take recourse to financial bequests at exactly the same rate of return as the return to education. At intermediate levels of initial wealth, symmetry-breaking results in the temporary upsurge of inequality, which is then reversed over time by a process of trickle-down: the process resembles a Kuznets curve.

In summary, we obtain a novel connection between initial poverty and subsequent disequalization. More generally, initial conditions matter, something that is common to a much larger literature. What is different is the *particular* set of initial conditions for inequality to emerge and persist, which can now include cases of perfect equality as well.

The two key assumptions that drive the analysis are the combination of a capital market imperfection and the essentiality of both skilled and unskilled workers in the production process. The former ensures that the acquired skills of the next generation will depend on parental wealths. The latter restriction then implies that in the presence of poverty, skilled wages will be high relative to unskilled wages. Hence a society which is sufficiently poor (but equal) in one generation will experience high inequality in the next generation, and this will subsequently become entrenched.

Modifications of the model that preserve these two central features will therefore be expected to give rise to similar results. For instance, if parents have a warm-glow bequest motive (as in Galor and Zeira (1993) or Banerjee and Newman (1993)), where they care about the size of the bequest rather than its consequences for their children, symmetry-breaking will not occur at the parental decision stage. But it will occur later, when educational decisions are made.[8] This observation

[8]To be sure, if educational decisions are also based on warm-glow considerations *alone*, there will be no market effects on the demand for education, but we exclude this rather extreme case. The model may also generate situations where sufficient poverty combined with equality in one generation generates bequests which are uniformly low so nobody in the next generation can afford

is robust to a variety of other bequest motives, including those in which parents want to be looked after in their old age, or educate their children in part to ensure that their progeny not fritter away their finances in idle pursuits. What is important is that education should be sensitive to the market differential in wages, so that symmetry-breaking can occur.

The particular source of capital market imperfection is also not crucial to our analysis. We assumed children cannot take on the debts of their parents. In less developed countries unpaid debts often lead to situations of bonded labor for borrowers and their children. However, our results will continue to apply as long as children who are bonded cannot obtain a college education, and parents feel anxious about this possible outcome for their children. Other sources of capital market imperfections, such as problems in ensuring repayment of loans, will also give rise to the same kind of results.

Our model applies to indivisible investments such as those involved in forms of higher education or professional qualifications, rather than years of primary or secondary schooling. In a related paper (Mookherjee and Ray (2009)), we explore the steady states of a model with continuous education choices. If all occupations are essential, there is a unique steady state, which is either equal or unequal depending on how wide the span of training costs across different occupations is relative to the extent of parental altruism. Hence the question posed in this paper cannot arise in that context. Non-steady-state dynamics in that context is also far more complicated, and nothing is known about convergence properties of competitive equilibria.

Finally, in this paper we assumed that there is international movement of financial capital, so that the rate of return on financial wealth is fixed. In a closed economy, the stock of physical capital would determine the rate of return, and the analysis needs to be conducted using different methods. The potential usefulness of such an analysis is that it would allow a study of the effects of financial globalization (where the economy goes from being closed to having perfect capital mobility, which is the case in this paper).

education. The analysis will then have to modified slightly (e.g., with high-cost borrowing opportunities, or heterogeneity of learning abilities among children that there will always be sufficiently intelligent children who will acquire education with minimal resources, as in Mookherjee and Napel (2007)).

Appendix

Proof of Proposition 3. We begin by noting properties of competitive equilibrium in any given generation t, with a given distribution of inherited wealth $F_t(W)$ satisfying $F_t(x) < 1$.

LEMMA **1.** *Every competitive equilibrium (CE) in generation t will give rise to a skill ratio in the next generation satisfying $\lambda_{t+1} \leq \lambda^*$.*

Proof. If this is false there is a CE skill ratio $\lambda_{t+1} > \lambda^*$. Then the rate of return to investing in education is less than r, the rate of return to investing in financial capital, and nobody will want to acquire education in generation $t+1$, i.e., $\lambda_{t+1} = 0$, a contradiction. □

LEMMA **2.** *A necessary and sufficient condition for existence of a CE skill ratio $\lambda_{t+1} = \lambda^*$ is that*

$$1 - F_t(W^*) \geq \lambda^* \tag{7}$$

where W^* is the wealth of a parent in a hypothetical Becker-Tomes finance-only world with constant flow wage $w_u(\lambda^*)$ who leaves a bequest of precisely x. In this case there is no other CE skill ratio in generation $t+1$.

Proof. First suppose (7) holds. We first show there is a CE with skill ratio $\lambda_{t+1} = \lambda^*$. By definition, households with wealth at least W^* are willing to bequeath at least x when $\lambda_{t+1} = \lambda^*$, since the return (in the form of child's wealth) to bequests is the same as in the hypothetical finance-only world with flow wage $w_u(\lambda^*)$. In this world parents and children are indifferent between education and financial investments. So we can select λ^* households from this group (with parents wealth at least W^*), and require them to invest in education, while all the rest of the households in the economy do not invest in education. Then each household is behaving optimally and we have a competitive equilibrium.

We claim this is the only CE skill ratio in generation $t+1$. Otherwise by Lemma 1 there is a CE with $\lambda_{t+1} < \lambda^*$. Then the rate of return to education is higher than r, which will increase the incentive to invest in education of all those households with wealth at least W^* compared to the situation where $\lambda_{t+1} = \lambda^*$. Hence $\lambda_{t+1} \geq \lambda^*$, a contradiction.

The converse is obvious: if there is a CE skill ratio $\lambda_{t+1} = \lambda^*$, there must be at least λ^* parents willing to invest at least W^* when the rate of return on education equals r, which is exactly condition (7). □

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LEMMA **3.** *If (7) does not hold, there is a unique CE skill ratio which satisfies $\lambda_{t+1} < \lambda^*$. The equilibrium is characterized by a wealth threshold W_t (satisfying $\lambda_{t+1} = 1 - F_t(W_t)$) such that an unskilled parent with wealth at this threshold is indifferent between bequeathing just enough (x) to enable her child to get educated, and bequeathing less, in the sense that:*

$$U(W_t - x) + V(w_s(\lambda_{t+1})) = Z(W_t, w_u(\lambda_{t+1})). \tag{8}$$

In this case $F_{t+1}(W_t) < F_t(W_t) = 1 - \lambda_{t+1}$ implies $\lambda_{t+2} > \lambda_{t+1}$. Otherwise $\lambda_{t+2} = \lambda_{t+1}$.

Proof. By the previous Lemmas, a CE must involve $\lambda_{t+1} < \lambda^*$ if (7) does not hold. We now show that such a CE exists and is unique.

Take any skill ratio $\lambda < \lambda^*$ and suppose this is anticipated to prevail in generation $t+1$ by generation t parents. Then the child's wealth $\mu(b)$ as a function of bequest b equals $w_u(\lambda) + (1+r)b$ if $b < x$, and $w_s(\lambda) + (1 + r(b - x))$ otherwise. Hence there is a discontinuous upward jump in child's future wealth at $b = x$.

Consider the incentive of parents in generation t to leave bequests to their children as a function of their own wealth W: they will select b to maximize $U(W - b) + V(\mu(b))$. It is evident from the concavity and monotonicity of U and V that there exists a threshold wealth $\tilde{W}$ such that all parents with wealth at least $\tilde{W}$ will bequeath at least x, and those with wealth below this threshold will bequeath less than x. And the children of the former set will acquire skill at $t+1$, while those of the latter set will not. Hence the skill ratio that will result at $t+1$ equals the proportion of parents at t with wealth above the threshold $\tilde{W}$.

An increase in the anticipated skill ratio λ in generation $t+1$ will lower parental incentives to bequeath to their children, in the sense that $\tilde{W}$ will increase. This implies the actual skill ratio at $t+1$ will decline. Hence (given the Inada conditions) there is a unique skill ratio λ_{t+1} which if anticipated will be subsequently realized. Denote the threshold wealth $\tilde{W}$ corresponding to this skill ratio by W_t which then satisfies (8).

Now suppose $F_{t+1}(W_t) < F_t(W_t)$. If $\lambda_{t+2} < \lambda_{t+1}$ then every parent at $t+1$ with wealth at least W_t will want to bequeath at least x, since they want to do so if they were to anticipate a skill ratio of λ_{t+1} to prevail at $t+2$. By hypothesis there are more families with wealth at least W_t in generation $t+1$ than in generation t. Hence $\lambda_{t+2} \geq \lambda_{t+1}$, a contradiction.

Finally, suppose $F_{t+1}(W_t) \geq F_t(W_t)$. Then we cannot have $\lambda_{t+2} > \lambda_{t+1}$, as this would imply that a household with wealth W_t would strictly prefer not to bequeath at least x at $t+1$. By hypothesis, there are more households poorer than W_t at $t+1$ than t. Hence we must have $\lambda_{t+2} \leq \lambda_{t+1}$, a contradiction. So $\lambda_{t+2} \leq \lambda_{t+1}$. Now if $\lambda_{t+2} < \lambda_{t+1}$ then every skilled family will want to invest in education at $t+1$

(since they want to do so even at λ^*), and there are λ_{t+1} skilled families at $t+1$, implying that $\lambda_{t+2} \geq \lambda_{t+1}$, a contradiction. Hence $\lambda_{t+2} = \lambda_{t+1}$.

This concludes the proof of Lemma 3. □

Note that there is no monotone structure on the set of unequal steady state skill ratio, because increases in λ lower the cost of investing in education for the unskilled (as they become richer), and also the benefit of education. In general, therefore, Γ is the union of intervals $[\lambda^i, \lambda^{i+1}]$, with $i = 0, 2, 4, \ldots$. Condition (6) is satisfied as an equality at each λ^i, as a strict inequality in every λ in $(\lambda^i, \lambda^{i+1})$ with i even, and is violated in every λ in $(\lambda^i, \lambda^{i+1})$ with i odd.

Before we proceed to the dynamics, we need the following notation. Let W^i denote $\Omega(w_u(\lambda^i))$, the steady state wealth of the unskilled at the boundary unequal steady state λ^i.

LEMMA **4.** *A CE skill ratio $\lambda_{t+1} < \lambda^*$ and associated wealth threshold W_t as defined in (8) satisfies the following properties:*
(a) $W_t = W^i$ *implies* $\lambda_{t+1} = \lambda^i$.

(b) $W_t \in (W^i, W^{i+1})$ *with i even implies* $\lambda_t \in (\lambda^i, \lambda^{i+1})$ *and* $W_t \geq \Omega(w_u(\lambda_{t+1}))$.

(c) $W_t \in (W^i, W^{i+1})$ *with i odd implies* $\lambda_t \in (\lambda^i, \lambda^{i+1})$ *and* $W_t < \Omega(w_u(\lambda_{t+1}))$.

Proof. Recall condition (8) relating the threshold wealth W_t with the competitive equilibrium skill ratio λ_{t+1}. Compare this with the relation between steady state wealth W^i and skill ratio at any boundary (unequal) steady state skill ratio λ^i:

$$Z(W^i, w_u(\lambda^i)) = U(W^i - x) + V(w_s(\lambda^i)). \tag{9}$$

Part (a) follows from comparing these two conditions. For part (b), note that $W_t > W^i$ implies that anticipating the skill ratio λ^i at $t+1$, the threshold wealth type W_t would strictly prefer to bequeath at least x, so $\lambda_{t+1} > \lambda^i$. Conversely this type would prefer not to bequeath at least x upon anticipating a skill ratio of λ^{i+1}, so $\lambda_{t+1} < \lambda^{i+1}$. Hence λ_{t+1} is an unequal steady state ratio, with

$$\begin{aligned} Z(\Omega(w_u(\lambda_{t+1})), w_u(\lambda_{t+1})) &\geq \max_{b \geq 0} [U(\Omega(w_u(\lambda_{t+1})) - x - b) + V(w_s(\lambda_{t+1}) + b(1+r))] \\ &\geq U(\Omega(w_u(\lambda_{t+1})) - x) + V(w_s(\lambda_{t+1}))]. \end{aligned}$$

Now compare with (8) to infer that $W_t \geq \Omega(w_u(\lambda_{t+1}))$.

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Next turn to part (c). The same argument as for part (b) shows that $\lambda_{t+1} \in (\lambda^i, \lambda^{i+1})$. Since i is now odd, λ_{t+1} is not a (unequal) steady state skill ratio. Moreover $\lambda_{t+1} < \lambda^{i+1} < \lambda^*$. Proposition 2 now implies

$$Z(\Omega(w_u(\lambda_{t+1})), w_u(\lambda_{t+1})) < \max_{b \geq 0}[U(\Omega(w_u(\lambda_{t+1})) - x - b) + V(w_s(\lambda_{t+1}) + b(1+r))]. \tag{10}$$

We claim that this implies

$$Z(\Omega(w_u(\lambda_{t+1})), w_u(\lambda_{t+1})) < [U(\Omega(w_u(\lambda_{t+1})) - x) + V(w_s(\lambda_{t+1}))]. \tag{11}$$

Otherwise the maximum on the right hand side of (10) is attained at some positive b: a household with wealth $\Omega(w_u(\lambda_{t+1}))$ prefers to bequeath more than x. The convexity of preferences then implies that a pure educational investment (i.e., a bequest of x) would in turn dominate any bequest less than x, contradicting the hypothesis. Finally the result that $W_t < \Omega(w_u(\lambda_{t+1}))$ follows upon comparing (8) with (11). This concludes the proof of Lemma 4. □

To complete the proof of Proposition 3, consider first case (b), in which $\lambda_{t+1} \in (\lambda^i, \lambda^{i+1})$. Lemma 4 shows in this case that $W_t \geq \Omega(w_u(\lambda_{t+1}))$. Then all unskilled households at t (i.e., whose parents had wealth below W_t at t) will also have wealth below W_t at $t+1$. The reason is that those with wealth between W_t and $\Omega(w_u(\lambda_{t+1}))$ will leave less to their children. And those at or below $\Omega(w_u(\lambda_{t+1}))$ will leave more than they themselves inherited, yet their children's wealth cannot exceed $\Omega(w_u(\lambda_{t+1}))$. So the mass of the wealth distribution below W_t is not smaller at $t+1$ than at t. The last part of Lemma 3 now implies that $\lambda_{t+2} = \lambda_{t+1}$. In turn this implies that $W_{t+2} = W_{t+1}$. So the same story applies at $t+1$ as at t. The equilibrium skill ratio will remain stationary at λ_{t+1} for all $T \geq t+1$. Along this process, the wealth of the unskilled will converge to $\Omega(w_u(\lambda_{t+1}))$, while those of the skilled will converge to $\Omega(w_s(\lambda_{t+1}))$, so the economy converges to the unequal steady state associated with skill ratio λ_{t+1}.

Next consider case (c), in which $\lambda_t \in (\lambda^i, \lambda^{i+1})$ with i odd and $W_t < \Omega(w_u(\lambda_{t+1}))$. Now the wealth of all unskilled households rises towards $\Omega(w_u(\lambda_{t+1}))$. It is still possible that $W_{t+1} = W_t$ and thus $\lambda_{t+1} = \lambda_t$, but if so their wealths will move even closer to $\Omega(w_u(\lambda_{t+1}))$. Eventually at some future generation $t+T$, we must have $F_{t+T}(W_t) < F_t(W_t)$. Then Lemma 3 implies that $\lambda_{t+T} > \lambda_t$. But comparing with condition (9) applied to $i+1$, it follows that $\lambda_{t+T} < \lambda^{i+1}$. Applying the same argument from $t+T$ onwards, it follows that the equilibrium skill ratio is a nondecreasing sequence bounded above by λ^{i+1}. So it must converge. It can only converge to a steady state skill ratio, which must therefore be λ^{i+1}.

Finally consider the case where $\lambda_{t+1} > \bar{\lambda}$, in which case $W_t > \Omega(w_u(\bar{\lambda}))$. Then also Lemma 4 implies $W_t < \Omega(w_u(\lambda_{t+1}))$. The same logic as in case (c) now implies the equilibrium skill ratio is a monotone sequence, bounded above by the equal steady state ratio λ^*. So it must converge, and to a steady state skill ratio. Since the only steady state skill ratio above λ_{t+1} is λ^*, this is the skill ratio it must converge to. This completes the proof of Proposition 3. Proposition 4 is a straightforward corollary.

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