

POVERTY AND SELF-CONTROL

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We argue that poverty can perpetuate itself by undermining the capacity for self-control. In line with a distinguished psychological literature, we consider modes of self-control that involve the self-imposed use of contingent punishments and rewards. We study settings in which consumers with quasi-hyperbolic preferences confront an otherwise standard intertemporal allocation problem with credit constraints. Our main result demonstrates that low initial assets can limit self-control, trapping people in poverty, while individuals with high initial assets can accumulate indefinitely. Thus, even temporary policies that *initiate* accumulation among the poor may be effective. We examine implications concerning the effect of access to credit on saving, the demand for commitment devices, the design of financial accounts to promote accumulation, and the variation of the marginal propensity to consume across income from different sources. We also explore the nature of optimal self-control, demonstrating that it has a simple and behaviorally plausible structure that is immune to self-renegotiation.

KEYWORDS: Poverty, self-control, time inconsistency.

“When you ain’t got nothin’, you got nothin’ to lose.” Bob Dylan

1. INTRODUCTION

RECENT RESEARCH INDICATES that the poor not only borrow at high rates,² but also forego profitable small investments.³ These behavioral patterns contribute to the persistence of poverty, particularly (but not exclusively) in developing countries. Traditional theory (based on high rates of discount, minimum

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²Informal interest rates in developing countries are notoriously high; see, for example, Aleem (1993). So are formal interest rates. For example, Bangladesh recently capped microfinance interest rates at 27% per annum, a restriction frowned upon by the *Economist* (“Leave Well Alone,” November 18, 2010). Citing other literature, Banerjee and Mullainathan (2010) argued that such loans are taken routinely rather than on an emergency basis.

³High returns have been documented for agricultural investments in Ghana, even on small plots (Goldstein and Udry (1999) and Udry and Anagol (2006)), the use of small amounts of fertilizer in Kenya (Duflo, Kremer, and Robinson (2011)), and microenterprise in Sri Lanka (de Mel, McKenzie, and Woodruff (2008)). See Banerjee and Duflo (2011) for additional references.

subsistence needs, and/or aspiration failures) can take us only part of the way to an explanation. Because the poor also exhibit a demand for commitment,⁴ it is likely that time inconsistency plays an important contributory role.

It is generally understood that time inconsistency can create self-control problems.⁵ We are interested in the possibility that difficult economic circumstances may exacerbate these problems. If self-control (or the lack thereof) is a fixed trait independent of economic circumstances, then the outlook for temporary policy interventions that encourage the poor to invest in their futures is not good. But if poverty perpetuates itself by impairing the ability of time-inconsistent decision makers to exercise self-control,⁶ even temporary policies that help the poor to *initiate* asset accumulation may be highly effective.

The term “self-control” can refer to the use of either internal psychological mechanisms or externally enforced commitment devices; here, we focus primarily on the former. The defining feature of the mechanisms we examine is that they involve the self-imposed use of contingent punishments and rewards to establish incentives for following desired plans of action. Abundant psychological foundations for such “contingent self-reinforcement” are found in the literatures on self-regulation and behavior modification, dating back to the 1960s.⁷ According to this literature, people “often set themselves relatively explicit criteria of achievement, failure to meet which is considered undeserving of self-reward and may elicit self-denial or even self-punitive responses. . .” (Bandura and Kupers (1964)).

Formally, we view intertemporal choice as a dynamic game played by successive incarnations of a single decision maker with quasi-hyperbolic preferences

⁴See, for example, Shipton (1992) on the use of lockboxes in Gambia, Thaler and Benartzi (2004) on employee commitments to save out of future wage increases in the United States, and Ashraf, Karlan, and Yin (2006) on the use of a commitment savings product in the Philippines. Aliber (2001), Gugerty (2007), and Anderson and Baland (2002) viewed ROSCA participation as a commitment device; see also the theoretical contributions of Ambec and Treich (2007) and Basu (2011). Duflo, Kremer, and Robinson (2011) attributed low fertilizer use in Kenya to a lack of commitment opportunities. Commitment issues also feature prominently in the ongoing debate over whether to replace the public system for distributing food in India with a cash-based program; see Khera (2011).

⁵See, for instance, Strotz (1955), Phelps and Pollak (1968), Ainslie (1975, 1991), Thaler and Shefrin (1981), Akerlof (1991), Laibson (1997), O’Donoghue and Rabin (1999), and Ashraf, Karlan, and Yin (2006). Such inconsistency may be internal to the individual, or have social origins stemming from discordance within the household (e.g., spouses with different discount factors) or from demands made by the wider community (e.g., sharing among kin).

⁶Aspiration failures can create similar traps. See, for example, Appadurai (2004), Ray (2006), Genicot and Ray (2014), and the *United Nations Development Program* Regional Report for Latin America, 2010. However, this complementary approach does not generate a demand for commitment devices.

⁷See, for example, Bandura and Kupers (1964), Bandura (1971, 1976), Mischel (1973), Rehm (1977), and Kazdin (2012). The Supplemental Material (Bernheim, Ray, and Yeltekin (2015)) reviews this literature in more detail.

(also known as $\beta\delta$ -discounting).⁸ In this setting, it is natural to equate contingent self-reinforcement with the use of *history-dependent* strategies. But there is a potential credibility problem: the consumer may not follow through on plans to impose punishments on (or withhold rewards from) himself. We resolve the credibility problem by insisting on subgame perfection. In short, we interpret subgame-perfect, history-dependent equilibrium strategies as methods of exercising self-control through the credible deployment of contingent punishment and reward.

While psychologists do not typically employ the language of game theory, they have long recognized that credibility problems can limit the efficacy of contingent self-reinforcement in achieving self-control.⁹ In fact, the logic of using history-dependent strategies to generate credibility is a recurring theme in Ainslie's (1975, 1991, 1992) work on "personal rules." He described the typical personal rule as "a solution to the bargaining problem" between an individual's "successive motivational states," through which that individual "can arrange consistent motivation" for a "prolonged course of action." Indeed, Ainslie (1991) informally constructed a subgame-perfect equilibrium in which an individual exercises self-discipline (going to bed early every night) by contriving conditional self-punishment (staying up late for ten consecutive nights), and likened the problem to "a repeated prisoner's dilemma."¹⁰ In contrast, economists studying hyperbolic discounting and time inconsistency have focused almost exclusively on Markov-perfect equilibria, which involve no history dependence, and hence cannot capture the phenomenon of contingent self-reinforcement.¹¹

We study a standard intertemporal allocation problem in which the individual faces a credit constraint. To avoid trivially building in (by assumption) a relationship between initial assets and the rate of saving, we take preferences to be homothetic and the accumulation technology to be linear. To determine the full scope for self-control, we study the set of *all* subgame-perfect Nash equilibria. This approach allows us to identify the conditions under which wealth accumulation can or cannot occur. In particular, we ask whether self-control is more difficult when initial assets are low than when they are high.

It is notoriously difficult to characterize the set of subgame-perfect Nash equilibria for all but the simplest dynamic games. The problem of self-control

⁸Strotz (1955) pioneered this general approach. Models of quasi-hyperbolic discounting were popularized by Laibson (1994, 1996, 1997) and O'Donoghue and Rabin (1999).

⁹Ainslie (1975) succinctly summarized the problem thus: "Self-reward is an intuitively pleasing strategy until one asks how the self-rewarding behavior is itself controlled..." See also Rachlin (1974) and Kazdin (2012).

¹⁰The Supplemental Material contains a more complete account of this literature. We also contrast our interpretation of Ainslie's personal rules as history-dependent strategies with other possible readings of his work.

¹¹We expand further on this point in Section 6. Exceptions to the use of Markov equilibrium include Laibson (1994), Bernheim, Ray, and Yeltekin (1999), and Benhabib and Bisin (2001).

we study here is, alas, no exception. Obviously, for some parameter values, equilibrium accumulation will either *always* occur or *never* occur, regardless of initial assets. Yet there are also parameter values (with intermediate degrees of time inconsistency) for which equilibrium accumulation will depend on the initial asset level. Our main result demonstrates that, in every such case, there is an asset level below which liquid wealth is unavoidably exhausted in finite time (a poverty trap), as well as an asset level above which unbounded accumulation is feasible.¹² Thus, low initial wealth precludes self-control, while high initial wealth permits it.¹³

Figure 1 illustrates our main result computationally.¹⁴ Horizontal axes measure current assets. The vertical axis in panel A measures continuation asset choices for the next period. Points above, on, and below the 45° line indicate asset accumulation, maintenance, and decumulation, respectively. Panel B

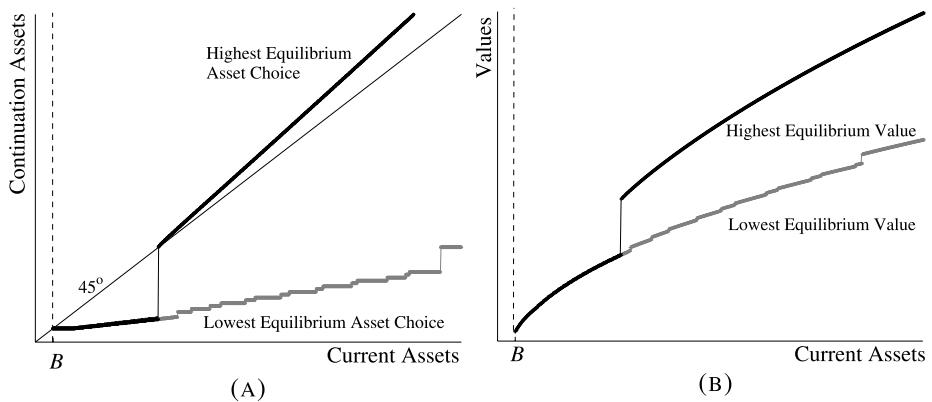


FIGURE 1.—Accumulation and values at different asset levels. (A) Equilibrium asset choices. (B) Equilibrium values.

¹²According to the δ -discounting criterion, the most attractive equilibria involve unbounded accumulation. It follows that, with $\beta\delta$ -discounting, the individual always aspires to accumulate wealth in the future.

¹³Notice that high initial wealth does not *necessitate* self-control; it enables the mechanism of interest, but it does not necessarily activate that mechanism. We consider this feature of our theory a virtue: it implies that some (but not all) people who suffer from impulsiveness may be able to learn effective methods of self-regulation (in effect, a new equilibrium). Such learning is implicit in Kazdin's (2012) observation that "[s]elf-reinforcement and self-punishment techniques have been incorporated into intervention programs and applied to a wide range of problems..."

¹⁴For a complete explanation of our computational methods, and for details concerning all computational examples presented in the text, see the Supplemental Material. For this exercise, we set the rate of return equal to 30%, the discount factor equal to 0.8, the hyperbolic parameter (β) equal to 0.4, and the constant elasticity parameter of the utility function equal to 0.5. We chose these values so that the interesting features of the equilibrium set are easily visible; qualitatively similar features arise for more realistic parameter values.

records the corresponding value function, in other words, the continuation value for a consumer with quasi-hyperbolic preferences, which equals the present value according to delta discounting. The example exhibits a poverty trap; that is, an asset threshold below which *all* equilibria lead to decumulation. However, above that threshold, asset accumulation is possible for some equilibria. Indeed, an equilibrium with the highest value generates unbounded accumulation, starting from above this threshold.¹⁵

A natural and intuitive explanation for this result is that credible self-punishments in the form of high future consumption become relatively more severe as assets accumulate. Intuitively, the decision maker cannot engage in high consumption if she is up against her credit constraint, but she has a great deal to lose if she possesses substantial wealth. It turns out, however, that the problem is considerably more complicated than this simple intuition suggests,¹⁶ because the credit constraint infects feasible behavior (and hence worst punishments) at *all* asset levels in subtle ways. The example in Figure 1 illustrates this point: there are asset levels at which the *lowest* level of continuation assets jumps up discontinuously. As assets cross those thresholds, the worst punishment becomes *less* rather than *more* severe. Note that the lowest equilibrium value shown in panel B of the figure, which serves as the worst punishment, jumps upward at several asset levels.¹⁷ To prove our main result, we show that at high asset levels, self-punishments become sufficiently severe to sustain accumulation, but this does not happen monotonically.

One might object to the entire exercise on the ground that subgame-perfect equilibria with history-dependent strategies involve unreasonably complex patterns of behavior. Yet we show that the worst credible punishments involve a simple, intuitive, and behaviorally plausible pattern of self-reinforcement. In effect, the individual sets a personal standard of behavior, which specifies how much she should save. If she fails to meet this standard, she punishes herself. The punishment involves a temporary binge, which we prove cannot exceed two periods; once it ends, she rededicates herself to her best achievable personal standards. Thus, if she “falls off the wagon,” she soon climbs back on; she responds to a lapse by “getting it out of her system” so she can adhere to her standards. As discussed in Section 5.5, these punishments are also renegotiation-proof in the sense of Bernheim and Ray (1989) and Farrell and Maskin (1989). These are conceptually simple and attractive features, though

¹⁵This is a more subtle point that cannot be seen directly from Figure 1, though the figure is indicative. It is more subtle because repeated application of the highest continuation asset need not be an equilibrium, and moreover, even if it were, it need not be the most attractive equilibrium.

¹⁶The overwhelmingly numerical nature of our earlier working paper, Bernheim, Ray, and Yeltekin (1999), bears witness to this assertion.

¹⁷The jagged nature of the lowest value in panel B is not a numerical artifact; it reflects actual jumps.

the precise computational details can be complex. But the same computational complexity appears for *all* equilibria in our model.¹⁸

Our analysis has a number of provocative implications for economic behavior and public policy. First, the relationship between assets and self-control argues for the use of “pump-priming” interventions that encourage the poor to start saving, while relying on self-control to sustain frugality at higher levels of assets. Second, policies that improve access to credit can help people become savers. Intuitively, with greater access to credit, the consequences of a break in discipline become more severe, and hence discipline is easier to sustain. But there is an important qualification: those who still cannot exercise self-control fall further into debt. Third, external commitment devices can undermine the effectiveness of internal self-control mechanisms. Consequently, when the latter are reasonably effective, people may avoid the former, even if they understand their self-control problems. Our theory therefore potentially accounts for the puzzling lack of demand for commitment devices observed in many contexts, particularly among the non-poor.¹⁹ Fourth, it may be possible to increase the effectiveness of incentives to save through special accounts (e.g., IRAs) by requiring the individual to establish a savings target, locking up all funds until the target is achieved, and then removing the lock (thereby rendering all of the funds liquid). Pilot programs with such features have indeed been tested in developing countries.²⁰ Finally, our analysis provides a potential explanation for the observation that the marginal propensity to consume differs across classes of resource claims, and offers a new perspective on the “excess sensitivity” of consumption to income.

Related Literature. We build on our unpublished working paper [Bernheim, Ray, and Yeltekin \(1999\)](#), which made its points through simulations, but did not contain analytical results. [Banerjee and Mullainathan \(2010\)](#) also argued that self-control problems give rise to low asset traps, but their analysis has little in common with ours. They examined a novel model of time-inconsistent preferences, in which rates of discount differ from one good to another. “Temptation goods” (those to which greater discount rates are applied) are presumed to be inferior goods: this *assumed* non-homotheticity of preferences automatically builds in a tendency to dissave when resources are limited. The validity of this central assumption (e.g., whether a poor person spends relatively more of his budget on alcohol than a richer person does on,

¹⁸Section 5.4 presents a simpler and more tractable version of our model and shows that higher levels of initial wealth are also more conducive to self-control in that setting. A possible interpretation of this result is that boundedly rational agents who attack the complex problem by thinking through simpler, more tractable versions will exhibit behavioral patterns that are qualitatively similar to those described in our main results.

¹⁹See, for example, [Bryan, Karlan, and Nelson \(2010\)](#) and [DellaVigna \(2009\)](#).

²⁰See [Ashraf, Karlan, and Yin \(2006\)](#), as well as [Karlan, McConnell, Mullainathan, and Zinman \(2010\)](#).

say, designer drugs or iPads) is plainly an empirical matter. Our approach relies on no such assumption; indeed, we adopt a standard model of time inconsistency in which preferences are homothetic and the accumulation technology linear. Scale effects arise only from the interplay between credit constraints and equilibrium conditions.²¹ Consequently, our analysis is essentially orthogonal (and hence potentially complementary) to that of [Banerjee and Mullainathan \(2010\)](#): theirs is driven by assumed scaling effects in rewards, while ours is driven by scaling effects in punishments arising from assumed credit market imperfections.²²

Organization of the Paper. Section 2 describes the model and defines equilibrium. Section 3 provides a characterization of the equilibrium value set that is useful both conceptually and in developing a numerical algorithm. Section 4 defines self-control, and Section 5 studies its relationship to initial wealth. Section 6 discusses Markov equilibrium. Section 7 explores additional implications. Section 8 presents conclusions and some directions for future research. An extended discussion of the relevant psychology literature, technical details for numerical computations, and (because of their length) formal proofs appear in the Supplemental Material.

2. MODEL

2.1. Feasible Set and Preferences

Current assets, current consumption, and future assets, starting from an initial asset level A_0 , are linked by

$$(1) \quad c_t = A_t - (A_{t+1}/\alpha) \geq 0,$$

where $\alpha - 1 > 0$ is the rate of return. Assets must also respect a lower bound:

$$(2) \quad A_t \geq B > 0.$$

We interpret B as a credit constraint.²³ For instance, if the individual earns a constant income y , then $A_t = F_t + [\alpha y/(\alpha - 1)]$, where F_t is financial wealth. If she can borrow only some fraction $(1 - \lambda)$ of future income, then $B = \lambda \alpha y/(\alpha - 1)$.

²¹Moav and Neeman (2012) analyzed a model with homothetic preferences in which conspicuous consumption generates poverty traps.

²²Our analysis is also related to that of Laibson (1994) and Benhabib and Bisin (2001), both of whom examined a model similar to ours, except that there is no credit constraint. Because the resulting model is fully scalable, so is the equilibrium set; consequently, there is no relationship between poverty and self-control.

²³Another interpretation of B is that it is an investment in an illiquid asset. We return to this interpretation when we discuss policy implications.

Individuals have quasi-hyperbolic preferences. Lifetime utility is given by

$$u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t),$$

where $\beta \in (0, 1)$ and $\delta \in (0, 1)$. We assume that u has the constant-elasticity form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

for $\sigma > 0$, with the understanding that $\sigma = 1$ refers to the logarithmic case $u(c) = \ln c$. With a linear accumulation technology and homothetic preferences, all nontrivial scale effects must arise from the interplay between credit constraints and equilibrium conditions.

2.2. Restrictions on the Model

The *Ramsey program* from A is the path $\{A_t\}$ that maximizes

$$\sum_{t=0}^{\infty} \delta^t \frac{c_t^{1-\sigma}}{1-\sigma},$$

with initial stock $A_0 = A$. It is constructed without reference to the hyperbolic factor β . This program is well-defined provided utilities do not diverge, which we ensure by assuming that

$$(3) \quad \gamma \equiv \delta^{1/\sigma} \alpha^{(1-\sigma)/\sigma} < 1.$$

We presume throughout that the Ramsey program exhibits growth, which requires

$$(4) \quad \delta \alpha > 1.$$

Under (3) and (4), the value of the Ramsey program is finite, and $c_t = (1 - \gamma)A_t$, while assets grow exponentially: $A_{t+1} = A_0(\delta^{1/\sigma} \alpha^{1/\sigma})^t = A_0(\gamma \alpha)^t$.

For $\sigma \geq 1$, utility is unbounded below, and so it is possible to sustain virtually any outcome using punishments that impose zero consumption, or a progressively more punitive sequence of vanishingly small consumptions; see [Laibson \(1994\)](#). Such devices are contrived and unrealistic; we rule them out by assuming that consumption is bounded below at every asset level:

$$(5) \quad c_t \geq v A_t,$$

where v is some small but positive number. Formally, it is enough to take $v < 1 - \gamma$, so that Ramsey accumulation is feasible, but we think of v as tiny. We

assume the lower bound on consumption is proportional to assets so as to avoid introducing artificial scale effects.

2.3. Equilibrium

Continuation asset A' is *feasible* given A if $A' \in [B, \alpha(1 - v)A]$. A *history* h_t at date t is a feasible sequence of assets (A_0, \dots, A_t) through date t . A *policy* ϕ specifies a feasible continuation asset $\phi(h_t)$ following every history. If the history h_t is followed by the asset choice x , we write the resulting history as $h_t.x$. A policy ϕ yields a *value* V_ϕ , as follows:

$$V_\phi(h_t) \equiv \sum_{s=t}^{\infty} \delta^{s-t} u\left(A(h_s) - \frac{\phi(h_s)}{\alpha}\right),$$

where $A(h_t)$ denotes the last element of h_t and, recursively, $h_{s+1} = h_s.\phi(h_s)$ for $s \geq t$. Similarly, ϕ also yields a *payoff* P_ϕ :

$$P_\phi(h_t) \equiv u\left(A(h_t) - \frac{\phi(h_t)}{\alpha}\right) + \beta\delta V_\phi(h_t.\phi(h_t)).$$

Note that payoffs include the hyperbolic factor β , while values do not.

An *equilibrium* is a policy such that, at every history h_t and for each x feasible given $A(h_t)$,

$$(6) \quad P_\phi(h_t) \geq u\left(A(h_t) - \frac{x}{\alpha}\right) + \beta\delta V_\phi(h_t.x).$$

Notice how the “one-shot deviation principle” is embedded in this definition. This is a natural corollary of the many-self perspective that we adopt, where in each period a different self is viewed as a fresh player. One can also consider coordinated deviations among multiple selves. We address this issue in our discussion of renegotiation-proofness; see Section 5.5.

For some of our results, it will be useful to assume that the set of equilibrium continuation values is convex. We therefore suppose that, following any asset choice, the continuation plan can be chosen (if needed) using a public randomization device.²⁴ Generalizing our notation to encompass public randomization is routine; we skip the details for the sake of brevity.

²⁴Here, “public” randomization involves conditioning the continuation equilibrium on the realization of a random variable that the individual does not subsequently forget.

3. EXISTENCE AND CHARACTERIZATION OF EQUILIBRIUM

For each $A \geq B$, let $\mathcal{V}(A)$ be the set of all equilibrium values available at A . If $\mathcal{V}(A)$ is nonempty, let $H(A)$ and $L(A)$ be its supremum and infimum values. Under (3) and (5), it is obvious that

$$-\infty < L(A) \leq H(A) \leq R(A) < \infty,$$

where $R(A)$ is the Ramsey value. In fact, a tighter bound is available for worst values:

OBSERVATION 1: *Suppose that $\mathcal{V}(A)$ is nonempty for every $A \geq B$. Then*

$$(7) \quad L(A) \geq u\left(A - \frac{B}{\alpha}\right) + \delta L(B) \geq u\left(A - \frac{B}{\alpha}\right) + \frac{\delta}{1 - \delta} u\left(\frac{\alpha - 1}{\alpha} B\right).$$

Observation 1 establishes a baseline for iterating a self-generation map. To this end, consider a nonempty-valued correspondence \mathcal{W} on $[B, \infty)$ such that for all $A \geq B$,

$$(8) \quad \mathcal{W}(A) \subseteq \left[u\left(A - \frac{B}{\alpha}\right) + \frac{\delta}{1 - \delta} u\left(\frac{\alpha - 1}{\alpha} B\right), R(A) \right].$$

Say that \mathcal{W} supports the value w at asset level A if there is a feasible asset choice x and $V \in \mathcal{W}(x)$ —a continuation $\{x, V\}$, in short—with

$$(9) \quad w = u\left(A - \frac{x}{\alpha}\right) + \delta V,$$

while for every feasible continuation asset x' ,

$$(10) \quad u\left(A - \frac{x}{\alpha}\right) + \beta \delta V \geq u\left(A - \frac{x'}{\alpha}\right) + \beta \delta V',$$

for some $V' \in \mathcal{W}(x')$. That is, the value w at A is “incentive-compatible” given suitable choices of continuation values from \mathcal{W} . Now say that \mathcal{W} generates the correspondence \mathcal{W}' if, for every $A \geq B$, $\mathcal{W}'(A)$ is the convex hull of all values supported at A by \mathcal{W} . (The convex hull captures public randomization; i.e., an asset choice can yield a lottery over continuation values.)

Standard arguments tell us that the equilibrium correspondence \mathcal{V} generates itself, and indeed it contains any other correspondence that does so. Accordingly, define a sequence of correspondences on $[B, \infty)$, $\{\mathcal{V}_k\}$, by

$$\mathcal{V}_0(A) = \left[u\left(A - \frac{B}{\alpha}\right) + \frac{\delta}{1 - \delta} u\left(\frac{\alpha - 1}{\alpha} B\right), R(A) \right]$$

for every $A \geq B$, and recursively, \mathcal{V}_k generates \mathcal{V}_{k+1} for all $k \geq 0$. It is obvious that the graph of \mathcal{V}_k contains the graph of \mathcal{V}_{k+1} . We assert the following:

PROPOSITION 1: *An equilibrium exists from any initial asset level. The equilibrium correspondence \mathcal{V} is nonempty-valued, convex-valued, and has closed graph. For every $A \geq B$,*

$$(11) \quad \mathcal{V}(A) = \bigcap_{k=0}^{\infty} \mathcal{V}_k(A).$$

The proof involves fairly standard arguments, though they must be adjusted for the fact that assets are unbounded. It also suggests an algorithm which we employ for numerical computation; see the Supplemental Material for details.²⁵ The proposition also shows that $L(A)$ and $H(A)$ are equilibrium values. In particular, L represents worst credible punishments.

4. SELF-CONTROL

Say that there is *self-control* at asset level A if the agent is capable of strict asset accumulation starting from A in some equilibrium. Say that there is *strong self-control* at A if the agent is capable of unbounded accumulation—that is, $A_t \rightarrow \infty$ —along some equilibrium path starting from A . Note that the definitions only tell us that equilibrium accumulation is an option, not that the individual will surely exercise that option.

In contrast, *self-control fails* at A if every equilibrium continuation asset level is strictly smaller than A , and more forcefully, that there is a *poverty trap* at A if, in every equilibrium, assets decline over time from A to the lower bound B . Unlike the notion of self-control, these definitions describe an inevitability rather than a possibility.

We call a case *uniform* if either (i) there is no asset level at which self-control is possible, or (ii) there is no asset level at which self-control fails. Case (i) arises with a sufficiently high degree of time inconsistency (β small), and case (ii) arises with β sufficiently close to 1 (because some equilibrium then approximates the Ramsey program, which involves indefinite accumulation by assumption). With perfect credit markets ($B = 0$), uniformity necessarily prevails: if continuation asset x can be sustained at A , then λx can be sustained at λA for any $\lambda > 0$. Consequently, there is no relationship between initial assets and the exercise of self-control.

²⁵Public randomization is not needed to establish existence; the same argument would work without it, except that \mathcal{V} would not generally be convex-valued. For related existence theorems encompassing other types of dynamic games with state variables, see Goldman (1980) and Harris (1985).

When $B > 0$, non-uniformity is possible. Indeed, we have computed the equilibrium correspondence for a large collection of parameter values satisfying our assumptions, and always find non-uniformity for intermediate values of β . (See the Supplemental Material.) With non-uniformity, the ability to exercise self-control varies with wealth; this case is of particular interest to us. The central issue we wish to explore is whether self-control is more prevalent at high or low asset levels.

Let $X(A)$ denote the highest level of continuation assets sustainable at A . By Proposition 1, $X(A)$ is well-defined and upper semicontinuous (u.s.c.). A single-crossing argument tells us that it is nondecreasing. Note that $X(A)$ is not necessarily the value-maximizing asset choice; it could be higher. The following intuitive result tells us that the function X completely characterizes self-control.

PROPOSITION 2: (i) *Self-control is possible at A if and only if $X(A) > A$.*

(ii) *Strong self-control is possible at A if and only if $X(A') > A'$ for all $A' \geq A$.*

(iii) *There is a poverty trap at A if and only if $X(A') < A'$ for all $A' \in (B, A]$.*

(iv) *There is uniformity if and only if $X(A) \geq A$ for all $A \geq B$, or $X(A) \leq A$ for all $A \geq B$.*

Parts (i) and (iv) are obvious. Part (iii) follows from the observations that X is nondecreasing and u.s.c. To prove part (ii), we consider the problem of maximizing $\sum_{t=0}^{\infty} \delta^t u(c_t)$ starting at some initial asset value A subject to feasibility ((1), (2), and (5)) plus the additional constraint that $A_{t+1} \leq X(A_t)$. Compared to another path that first chooses $X(A)$ and continues along a trajectory that yields $H(X(A))$ (which we know is sustainable), this constrained-value-maximizing path yields a weakly higher value *and* weakly higher first-period consumption, and so achieves a weakly higher payoff. Because this statement is true everywhere along our path, it is necessarily sustainable. Because the Ramsey path involves unbounded accumulation, so too must our path, provided $X(A') > A'$ for all $A' \geq A$. For details, see the Supplemental Material.

Our definition of self-control captures a fundamental descriptive feature of behavior: the ability or inability to accumulate assets. It does not rest on any particular normative perspective. At the same time, when part (ii) of Proposition 2 holds, the value-maximizing equilibrium does involve unbounded accumulation. That is noteworthy not only because it means the individual would like his future selves to accumulate, but also because it can be argued that value-maximization identifies the most attractive equilibrium from a welfare perspective.²⁶ While a normative foundation for our definition is therefore available, we do not pursue it here.

²⁶See Bernheim and Rangel (2009) for a discussion of formal justifications for using the “long-run” welfare criterion when evaluating welfare in the context of the quasi-hyperbolic model.

5. INITIAL ASSETS AND SELF-CONTROL

Why might asset levels affect an individual’s ability to exercise self-control? It is because that ability depends on the severity of the consequences that would follow an impetuous act. Intuitively, those consequences are potentially more severe when the individual has more assets, and hence more to lose. For instance, the individual cannot decumulate assets from B , but can do so from $A > B$. So it might be possible to accumulate assets by “threatening” decumulation as a contingent consequence starting from A , but not from B . Unfortunately, this appealing intuition oversimplifies the issues. As we have already seen (Figure 1, panel B), the worst credible self-punishment does not become monotonically worse (even with suitable renormalization) as assets grow. To understand the relationship between assets and self-control, we will need to uncover the structure of worst self-punishments.

5.1. Worst Self-Punishments

We will show that the worst self-punishments involve a temporary “binge,” followed by a return to the best continuation equilibrium value available at the asset level following the binge. Formally, for any $A > B$, let $H^-(A)$ be the left-hand limit of H at A (which is well-defined because H is nondecreasing; see Lemma 16 in the Supplemental Material).

PROPOSITION 3: (i) *The worst equilibrium value at any asset level A is implemented by choosing the smallest possible equilibrium continuation asset at A ; call it Y . If $Y > B$, the associated continuation value V satisfies $V \geq H^-(Y)$.*

(ii) *This equilibrium value can be generated by an equilibrium path, possibly with public randomization, which returns to the best continuation equilibrium after at most two periods.*

For an intuitive explanation of the bound on V in part (i), see Figure 2. Imagine that, following a deviation to continuation asset level A^d in period $t - 1$ (not shown in the figure), the equilibrium prescribes continuation asset level A' and a period- $(t + 1)$ continuation value $V' < H^-(A')$ (as shown in the figure). From the perspective of period t , this self-punishment involves a payoff of $\bar{P} \equiv u(A^d - \frac{A'}{\alpha}) + \beta\delta V'$ and a value of $\bar{V} \equiv u(A^d - \frac{A'}{\alpha}) + \delta V'$. The figure shows the iso-payoff curve $u(A^d - \frac{A'}{\alpha}) + \beta\delta V = \bar{P}$ and the iso-value curve $u(A^d - \frac{A'}{\alpha}) + \delta V = \bar{V}$. Both are upward-sloping (with higher payoff and value to the northwest), but the iso-value curve is flatter than the iso-payoff curve, because value places more weight on the future than payoff does. The iso-payoff curve necessarily remains above the $L(A)$ -curve: if it passed below L at some asset level A'' , the individual could profitably deviate by playing A'' rather than A' after choosing A^d . But then, provided $A' > B$, moving southwest along the iso-payoff curve both preserves incentive-compatibility and reduces value. Consequently, the worst self-punishment must lie at a point on

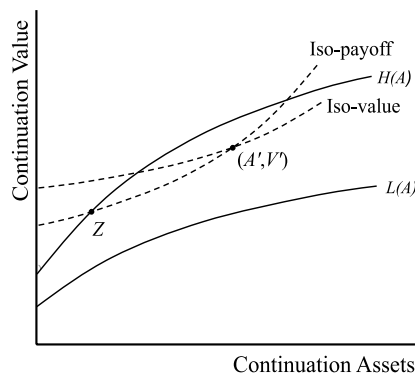


FIGURE 2.—Optimal self-punishment.

the upper envelope of the equilibrium value set, such as Z in the figure. A related argument shows that A' must be the lowest equilibrium continuation asset level.

Part (ii) of the proposition indicates how the individual can implement points on this envelope. We show that any such point is a convex combination of $H(Y)$ and the value of another equilibrium that implements the lowest continuation asset *again*, this time at Y , following up with the highest possible continuation value thereafter. This convex combination is achieved by publicly randomizing across the two equilibrium values. Unpacking the randomization, behavior looks like this: the individual binges for one, possibly two periods, then returns to the best possible continuation value.

Notice that the logic of Proposition 3 hinges on time inconsistency, which causes the iso-payoff and iso-value curves to diverge. Thus, while optimal self-punishments have a stick-and-carrot structure reminiscent of optimal penal codes for repeated games (Abreu (1988)), a version of that structure appears here for different reasons.

When worst self-punishments are used to support the choices that achieve $H(\cdot)$, the resulting equilibrium has a natural behavioral and psychological interpretation. The individual sets a personal standard of behavior, which prescribes a level of saving at each asset level A (and yields value $H(A)$). If she fails to meet this standard, she self-punishes. The punishment involves high consumption for one period, possibly two, which is disagreeable to the agent as viewed from the vantage point of her original deviation. But the proposition also states that she rededicates herself to her personal standards after at most two periods. In summary, a deviation causes our individual to “fall off the wagon,” after which she returns to making choices consistent with achieving her best personal standards, resulting in the value H from that point on.²⁷

²⁷It should be noted that the best standards may not be good enough, in the sense of achieving self-control. As Proposition 4 will show, this may be true of individuals with assets close to the

One reason why we are often skeptical of history-dependent strategies is that these could be highly complex. But the strategy we have identified has a qualitatively simple structure. It formalizes a natural fear—“if I deviate now, I will deviate later”—and yet it incorporates the comforting assurance that one can return to best practice in the future. In contrast, Markov punishments carry the unrealistic implication that self-punishments are permanent; see Section 6. From that perspective, optimal punishments surely do not suffer from lack of realism. That is not to say that a calculation of the exact extent of the binge, or indeed the best equilibrium post-binge, is *computationally* simple. But one encounters similar computational complexity when studying all dynamic equilibria in this context, including Markov equilibrium. (See also the discussion of complexity in Section 5.4.)

5.2. *The Relationship Between Wealth and Self-Control*

The possibility that an increase in wealth can render the worst (renormalized) self-punishment less severe leads to the nihilistic suspicion that there might be no general connection between wealth and self-control. Nevertheless, in every numerical example we have examined (see Supplemental Material), either self-control is uniform, or there is an asset threshold below which self-control is infeasible, resulting in a poverty trap, and above which self-control and unbounded accumulation become possible (as in Figure 1). One can certainly imagine other patterns—for example, that self-control is possible at low but not high asset levels, or that self-control recurs intermittently as assets grow. Indeed, Markov equilibria do exhibit a periodic structure with respect to self-control (see Section 6). Yet our main proposition rules these alternatives out:

PROPOSITION 4: *In any non-uniform case:*

- (i) *There is $A_1 > B$ such that every $A \in [B, A_1)$ exhibits a poverty trap.*
- (ii) *There is $A_2 \geq A_1$ such that every $A \geq A_2$ exhibits strong self-control.*

The next subsection informally sketches a proof of this proposition. For reasons of space, detailed arguments are relegated to the Supplemental Material.

5.3. *A Sketch of the Proof of the Main Proposition*

Although borrowing constraints destroy scale-neutrality of the equilibrium sets, variants of scale-neutrality survive. One useful variant appears below as Observation 2. To state it, we define an asset level $S \geq B$ as *sustainable* if there exists an equilibrium that permits indefinite maintenance of S . It is important

lower bound B , while for others with higher wealth a return to H will *also* mean a return to self-control. In both cases, Proposition 3 holds as stated.

to appreciate that strict accumulation may not be possible at a sustainable asset level, and more subtly, an asset level that permits strict accumulation need not be sustainable.²⁸

OBSERVATION 2: *Let $S > B$ be a sustainable asset level. Define $\mu \equiv S/B > 1$. Then for any initial asset level $A \geq B$, if continuation asset A' can be supported as an equilibrium choice, so can the continuation asset $\mu A'$ starting from μA .*

To understand this result, first think of S as a new lower bound on assets. Then, given the homotheticity of utility together with the linearity of the accumulation technology, Observation 2 would obviously hold. Because S is not actually a lower bound, we must also consider deviations to asset levels below S (which have no scaled-down counterparts when B is the lower bound). Lemma 8 in the Supplemental Material shows that the continuation $L(A)$ deters such deviations.

We use Observation 2 to prove part (i) of the proposition. Recall that $X(A)$ is the largest equilibrium continuation asset level at A . By Proposition 2, we need to show that there is an asset level $A_1 > B$ such that $X(A) < A$ for all $A \in (B, A_1)$. In the formal proof, we rule out the possibility that $X(A)$ wiggles back and forth across the 45° line ever more rapidly as $A \downarrow B$. With that in mind, if part (i) is false, there must be $M > B$ such that $X(A) \geq A$ for all $A \in [B, M]$. Figure 3 illustrates this case.

By non-uniformity, there is A^* at which self-control fails, so that $X(A^*) < A^*$ by Proposition 2. Let S be the supremum value of assets over $[B, A^*]$ for

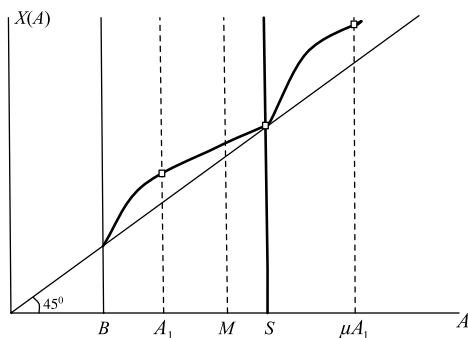


FIGURE 3.—Establishing the existence of a poverty trap.

²⁸With strict asset growth, the individual potentially has more to lose than with asset maintenance; hence the former may be easier to sustain.

which $A \in [B, S]$ implies $X(A) \geq A$. Clearly, $X(S) = S$,²⁹ which implies (by an additional argument) that S is sustainable.³⁰

Now Observation 2 implies that $X(A)$ must exceed A just to the right of S : for some A' close to B , just scale up $X(A')$ to $\mu X(A')$ at $\mu A'$, where $\mu \equiv S/B$. Because the definition of S requires $X(A) < A$ just to the right of S , we have a contradiction. It therefore follows that $X(A) < A$ for all A close to B , which means there is a poverty trap, as claimed.

Next, we explain part (ii) of the proposition. By non-uniformity, there is certainly some value of A for which $X(A) > A$. If the same inequality holds for all $A' > A$, then by Proposition 2(ii), strong self-control is established for A and all $A' > A$. So we need to address the case in which $X(A') \leq A'$ for some $A' > A$. Consider Figure 4, panel A. Focusing on the first zone over which $X(A) > A$, let S^* be the first asset level thereafter for which $X(A) = A$. As in our argument for part (i), S^* is sustainable.

By Observation 2, the choices $X(A)$ on $[B, S^*]$ can be scaled up and replicated as equilibrium choices on $[S^*, S_1]$, where S_1 bears the same ratio to S^* as does S^* to B .³¹ Figure 4 shows these choices as the dotted line within the domain $[S^*, S_1]$. Because there is a poverty trap near B , the dotted line lies be-

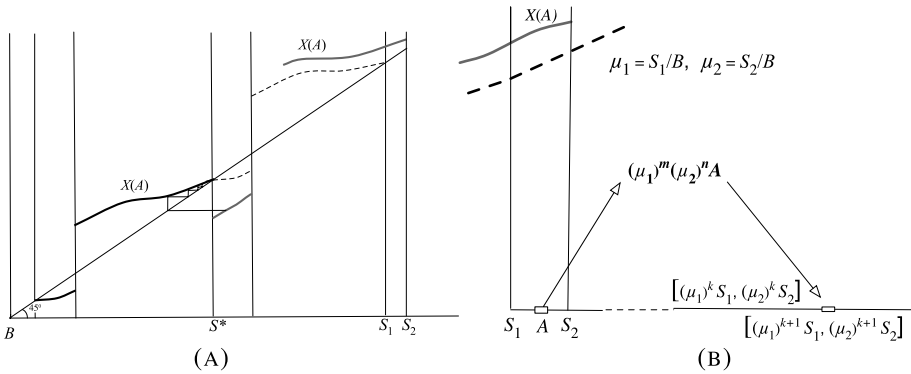


FIGURE 4.—Threshold for strong self-control. (A) Constructing the interval $[S_1, S_2]$. (B) Double scaling.

²⁹We cannot have $X(S) < S$, because X is nondecreasing, and we cannot have $X(S) > S$, because then S would not be the supremum of the set mentioned in the text.

³⁰In particular, the payoff from a constant asset trajectory is no smaller than one that remains constant for a single period and never subsequently exceeds its original level. Because incentive compatibility must hold for some trajectory of the second type (given $X(S) = S$), it must hold for the first.

³¹The actual proof becomes considerably more complex at this point. Briefly, the domain of interest is not exactly $[S^*, S_1]$, but an interval of the form $[S_{**}, S_1]$, where S_{**} might coincide with S^* but generally will not. (We proceed here on the assumption that S_{**} does coincide with S^* .) There are several associated complications, and the interested reader is referred to the Supplemental Material not just for the formalities, but also for further intuitive discussion.

low the 45° line just to the right of S^* . However—and this is at the heart of the argument below—that line does *not* coincide with $X(A)$ on $[S^*, S_1]$.

In the full proof, we show that worse punishments (in relative terms) are available near S^* than near B . Just to the right of S^* , one can construct equilibria that dip into the zone to the left of S^* , and then accumulate along $X(A)$ back toward S^* (as indicated by the “stairsteps” in the figure). Because these choices—shown by the solid line to the right of S^* in Figure 4—favor current consumption over the future, they generate even lower *values* than the equilibria that simply decline to S^* , but they earn high enough *payoffs* to be implementable. These lower values more effectively forestall deviations at even higher asset levels, and in this way greater punishment ability percolates upward from S^* . As a result, for asset levels close to S_1 , the incentive constraints are relaxed and higher levels of continuation assets are implementable (see the solid line segment in this region, which lies above the dotted line). In particular, in addition to being sustainable like S^* , S_1 also permits accumulation: $X(S_1) > S_1$.

This argument implies that there is an interval just above S_1 , call it (S_1, S_2) , over which (a) $X(A) > A$, and (b) both S_1 and S_2 are sustainable. Part (a) follows because $X(S_1) > S_1$ and X is nondecreasing. Part (b) follows from the fact that assets just to the right of S_1 were “almost sustainable” to begin with (by virtue of Observation 2); they become sustainable given the additional punishment power that percolates upward from S^* .

Panel B of Figure 4 focuses on the interval (S_1, S_2) and higher asset levels. The following property, stated and proven formally as Lemma 19 in the Supplemental Material, makes the key step:

OBSERVATION 3: *Suppose that S_1 and S_2 are both sustainable, and that $X(A) > A$ for all $A \in (S_1, S_2)$. Then there exists \hat{A} such that $X(A) > A$ for all $A > \hat{A}$.*

The proof of this observation is illustrated in panel B. Define $\mu_i = S_i/B$ for $i = 1, 2$. Then for all positive integers k larger than some threshold K , the intervals $(\mu_1^k S_1, \mu_2^k S_2)$ and $(\mu_1^{k+1} S_1, \mu_2^{k+1} S_2)$ *must overlap*. It is easy to see why: $\mu_2^k S_2$ is just $\mu_2^{k+1} B$, while $\mu_1^{k+1} S_1$ is $\mu_1^{k+2} B$, and for large k it must be that μ_2^{k+1} exceeds μ_1^{k+2} . Thus, we can generate any asset level $A > \mu_1^K S_1$ by simply choosing an integer $k \geq K$, an integer m between 0 and k , and $A' \in (S_1, S_2)$ so that $A = \mu_1^m \mu_2^{k-m} A'$. But $X(A') > A'$, so repeated application of Observation 2 proves that $X(A) > A$, which gives us Observation 3.

Part (ii) of Proposition 2 follows immediately: because $X(A) > A$ for all A sufficiently large, the required threshold A_2 must exist.

5.4. An Example

Proposition 4 is silent on a few counts. While there is ample numerical evidence that non-uniformity prevails for intermediate values of β , the propo-

sition does not show this. Neither does it establish the existence of a *unique* asset threshold above which self-control is possible and below which it is not—formally, a *single* point at which $X(A)$ crosses the 45° line. A demonstration of this stronger property is hindered in part by the possibility that worst punishments can move in unexpected ways as assets rise.³² From this perspective, the fact that after a finite threshold all such crossings must cease (as the proposition asserts) appears surprising. In addition, as even a casual perusal of Section 5.3 or the Supplemental Material will reveal, the proof is long and complex, which inevitably raises questions concerning behavioral plausibility. For these reasons, we examine a simplified version of the model, for which the result is sharper and the arguments quite elementary. This comparative simplicity is important, because it suggests that even simpler and more tractable versions of the model also have the implication that higher levels of initial wealth are more conducive to self-control.

Assume that *only two continuation asset choices are available*: a high level, given by $A_{t+1} = \lambda A_t$, or a low level, given by $A_{t+1} = \max\{B, A_t/\lambda\}$, for some given $\lambda \in (1, [\delta\alpha]^{1/\sigma})$. The upper bound on λ guarantees that accumulation involves no more consumption than the Ramsey path. In this setup, assets remain on the grid A^0, A^1, A^2, \dots , where $A^0 = B$ and $A^k = \lambda A^{k-1}$, provided they start there, and we suppose they do. Call this the “simplified model.”

We will also suppose that $\frac{\delta}{\lambda^{1-\sigma}} < 1$, so that discounted payoffs are well-defined on the decumulation path even when $B = 0$; notice that this assumption is automatically satisfied for $\sigma \leq 1$. It is easy to see (details in Supplemental Material) that in the class of feasible paths given by our restrictions, the value-maximizing path involves sustained accumulation in every period, while the value-minimizing path and hence the worst punishment involves sustained decumulation in every period.

PROPOSITION 5: *There exists $\bar{\lambda} \in (1, [\delta\alpha]^{1/\sigma})$ such that, for every $\lambda \in (1, \bar{\lambda})$, there is a nonempty interval (β_L, β_U) and a function w^* on (β_L, β_U) , with $w^*(\beta) > 1$ for all β , such that*

- (i) *if initial assets are smaller than $w^*(\beta)B$, the only equilibrium path in the simplified model involves sustained decumulation, and*
- (ii) *if initial assets exceed $w^*(\beta)B$, there exists an equilibrium path in the simplified model involving sustained accumulation.*

Furthermore, $w^(\beta)$ is increasing in β , and increases without bound as β approaches β_L .*

In the proposition, non-uniformity necessarily occurs for intermediate values of β . Moreover, in those cases, there is a *single* asset threshold below which a

³²However, despite an extensive search, we have not found a numerical example with multiple crossings.

poverty trap must exist, and above which unbounded accumulation is an equilibrium outcome. Notice that the threshold for the poverty trap is proportional to B . The same property holds in the general case, and we will return to its implications in Section 7. Finally, the proposition tells us that the asset threshold for viable self-control increases with the degree of time inconsistency, which is, of course, intuitive.

While we relegate algebraic details to the Supplemental Material, the argument of the proposition is straightforward. Consider whether it is possible to sustain an equilibrium with accumulation starting with wealth A^k . A necessary condition is that S_β^k , defined as the payoff from one-step accumulation followed by the highest-value feasible continuation path, is no less than D_β^k , defined as the payoff from one-step decumulation followed by the lowest-value feasible continuation path. That condition is sufficient if (a) the lowest-value feasible continuation path is also a Markov-perfect outcome, and hence a credible punishment, and (b) the condition is also satisfied for all $k' > k$ (so that the highest-value feasible continuation path is sustainable at all dates).

One can show that the inequality $S_\beta^k \geq D_\beta^k$ is equivalent to the condition $\beta \geq \beta_k$, where β_k is strictly positive and decreasing in k for $k \geq 1$. Intuitively, it is decreasing because the prospect of continual decumulation looks worse relative to the prospect of continual accumulation when initial wealth is greater. So the condition $\beta \geq \beta_k$ is easier to satisfy for larger k . Let $\beta_\infty \equiv \lim_{k \rightarrow \infty} \beta_k$. Because β_k decreases from β_1 to β_∞ , for any $\beta \in (\beta_\infty, \beta_1)$ there is $k^*(\beta) > 1$ such that $S_\beta^k \geq D_\beta^k$ is satisfied for all $k \geq k^*(\beta)$, but not for $k < k^*(\beta)$. Moreover, for lower value of β , $k^*(\beta)$ must be larger.

Taking $w^*(\beta) = \lambda^{k^*(\beta)}$, we are done save for one item: we have not yet verified that continual decumulation is a credible punishment. To complete the proof, we show that there is some $\beta_D > \beta_\infty$ such that, for $\beta \leq \beta_D$, decumulation from every initial asset is a Markov-perfect equilibrium. This is intuitive: β_D represents a threshold that (for very large k) makes immediate accumulation followed by a continually decumulating continuation path (i.e., $A^k, A^{k+1}, A^k, A^{k-1}, \dots$) yield the same payoff as continual decumulation ($A^k, A^{k-1}, A^{k-2}, \dots$). In contrast, β_∞ represents a threshold that (for very large k) makes sustained accumulation (i.e., $A^k, A^{k+1}, A^{k+2}, A^{k+3}, \dots$) yield the same payoff as continual decumulation. Under our assumptions, the continuation value is higher for continual accumulation than for continual decumulation, so a smaller value of β is required to establish indifference in the second instance.

Some of the richness of the full model is plainly lost when one adopts the simplifications described above. Most notably, optimal punishments no longer have the structure identified in Proposition 3. But Proposition 4 nevertheless

reassures us that the main message of Proposition 5 concerning poverty traps is not an artifact of simplifying assumptions.³³

5.5. Renegotiation-Proofness

A natural question is whether the equilibria we study are immune to renegotiation. Loosely, that term refers to the possibility that agents playing a game might choose to switch from the prevailing equilibrium, which embodies an ongoing self-enforcing agreement, to a more attractive one. In the context of equilibria that sustain cooperation in repeated games, a common concern is that players may have mutual incentives to renegotiate once they find themselves on unattractive punishment paths; hence the possibility of renegotiation may undermine useful cooperation.

One cannot formulate a theory of renegotiation without first specifying which agents take part in the negotiation. In some contexts, renegotiation may involve all of the players; in others, a subset may have the power to select and announce some new equilibrium, which all players then treat as the prevailing agreement. Similarly, one must also specify constraints on the negotiation. Plainly, the new agreement must be self-enforcing, and hence an equilibrium, but that alone is not a sufficient restriction: unless the new agreement is itself immune to future renegotiation, its announcement will have no force. In summary, then, we require (a) an understanding of the set of agents that might engage in renegotiation at any date, and (b) a description of the various plans over which they might negotiate.

Beginning with part (b), we follow [Bernheim and Ray \(1989\)](#) and [Farrell and Maskin \(1989\)](#): we consider the collection of all continuation paths for a given equilibrium and ask if it is internally consistent, in the sense that the renegotiating agents would never wish collectively to switch from one of these paths to another.³⁴ This requirement is known as *weak renegotiation-proofness* (WRP). It seems particularly appropriate in the present context if one thinks of the individual as inheriting behavioral principles (i.e., an equilibrium) by modeling mentors,³⁵ but retaining the ability to choose (and renegotiate) a starting point within that equilibrium.

³³A possible interpretation of this result, already noted in footnote 18, is that boundedly rational agents who impose manageable heuristics on the full, complex problem, such as those described here, will exhibit the same qualitative behavior as in Proposition 4. Viewed from that perspective, Proposition 5 shows that Proposition 4 is not an artifact of some excessively subtle consideration, and Proposition 4 shows that Proposition 5 is not simply an artifact of some ad hoc simplification.

³⁴[Bernheim and Ray \(1989\)](#) and [Farrell and Maskin \(1989\)](#) actually applied their concept to collections of equilibrium *payoffs*. Given the potentially confusing distinction between payoffs and values in the current setting, we apply the same notion to equilibrium paths or outcomes.

³⁵This possibility finds support in the psychological literature on self-reinforcement. For example, in a classic experiment, “children’s patterns and magnitude of self-reinforcement closely matched those of the model to whom they had been exposed. Adults generally served as more

To apply the WRP concept, we must specify an equilibrium, and hence the paths that are available to the negotiating parties. Consider an equilibrium that, absent any deviation, achieves the highest value H from all asset levels. Any such equilibrium exhibits accumulation whenever self-control is feasible, and unbounded accumulation whenever strong self-control is feasible (see Lemma 15 in the Supplemental Material). Therefore, if it is WRP, Proposition 4 extends to situations that permit renegotiation. To describe the equilibrium strategies, we consider two selections $X^*(A)$ and $Y^*(A)$ from the overall equilibrium asset choice correspondence, and two “plans” built from these selections. Plan 1 applies X^* repeatedly and generates the highest value $H(A)$. Plan 2, which is used to punish all deviations (whether from Plan 1 or Plan 2 itself), applies Y^* *once* at A to obtain the lowest possible equilibrium continuation asset $Y^*(A)$, and then uses a randomization device to switch back to Plan 1 at $Y^*(A)$, or restart Plan 2 at $Y^*(A)$. The randomization probability is chosen so that the expected value at $Y^*(A)$ is exactly equal to the continuation value of the worst punishment at A . By Proposition 3, Plan 2 implements the value $L(A)$ at A .³⁶ Hence it is easy to check that this is, in fact, an equilibrium.

Now for part (a). We examine several alternatives. Begin with the (reasonable) position that an agent must be present to take part in a renegotiation, which limits the set of negotiators to those present in period t . Here we consider two possibilities. The first of these embraces the view that the standard model of quasi-hyperbolic discounting is a reduced form for a process involving repeated Nash bargaining between two long-lived selves, an impulsive “doer” who cares only about current consumption, and a patient “planner” with standard Ramsey preferences, with the parameter β capturing the relative weight assigned to the patient self.³⁷ Under the WRP concept, our equilibrium set is vulnerable to renegotiation only if there is some history at which *both* the impulsive and patient selves would like to switch from one equilibrium plan to another. Yet that is never the case. Plan 1 provides a higher payoff to the patient self, while Plan 2 provides higher payoff to the impulsive self. Because the two selves do not agree, our equilibrium set is WRP. Thus consideration of this interpretation captures what is to us an appealing intuition: successful renegotiation in this setting is difficult because either the consumer’s impulsive side or her deliberative side will always resist.

The second possibility is to treat each date- t self as a single, distinct entity. “Renegotiation” then becomes a simple question of whether any date- t self

powerful modeling stimuli than peers in transmitting self-reinforcing responses” (Bandura and Kupers (1964); see also Bandura (1971, 1976)).

³⁶By Proposition 3, if H is continuous at Y and $Y > B$, the move back to Plan 1 is deterministic.

³⁷The doer-planner model was originally formulated by Thaler and Shefrin (1981); see also Fudenberg and Levine (2006). For the relationship to quasi-hyperbolic discounting, see Bernheim (2009).

ever prefers to switch between plans. According to the incentive constraints that uphold the equilibrium, we have (for Plans 1 and 2, respectively),

$$(12) \quad u\left(A - \frac{X^*(A)}{\alpha}\right) + \beta\delta H(X^*(A)) \geq D(A), \quad \text{and}$$

$$(13) \quad u\left(A - \frac{Y^*(A)}{\alpha}\right) + \beta\delta V \geq D(A),$$

where $D(A)$ is the “best” deviation payoff at A ,³⁸ and V is the continuation value associated with the worst punishment, obtained from future randomization between the values $H(Y^*(A))$ and $L(Y^*(A))$, as described above. Notice that, when both constraints bind (as they typically will at the extreme points of the equilibrium value correspondence), the date- t self is indifferent between the two plans, and therefore has no incentive to “renegotiate.” Once again, the equilibrium set is WRP. Here, the key insight is that the path used to punish a deviation by the $(t - 1)$ -self has the flavor of something the *punisher*—that is, the t -self—would like to follow, because it involves greater consumption at date t .

As a final alternative, suppose that renegotiation at date t encompasses future selves as well as the period- t self. When the set of negotiating parties expands (without altering the feasible set), successful renegotiation becomes more difficult. Accordingly, such considerations would strengthen our conclusions.

For an alternative perspective on renegotiation that is motivated by observations made in the psychological literature on contingent self-reinforcement, see our detailed discussion of psychological foundations in the Supplemental Material.

6. MARKOV EQUILIBRIUM

As we have mentioned, most of the literature on quasi-hyperbolic discounting focuses on Markov-perfect equilibria, in which choices depend only on the current asset level A , and not on how that level was reached (e.g., whether the individual exercised restraint from a lower asset level or splurged from a higher asset level). Because a Markov-perfect equilibrium provides no scope for *contingent* behavior, it cannot capture the self-regulatory phenomena described in the psychological literature.

That said, our formal definition of self-control is not specific to contingent self-reinforcement; it only concerns the ability to accumulate assets. It is there-

³⁸That is, $D(A)$ is the supremum over all payoffs in which every deviation to an alternative asset choice is “punished” by the lowest equilibrium value available at that asset. The function $D(A)$ is formally defined in the Supplemental Material, where we deal with various technicalities arising from lack of the continuity in the value correspondence.

fore of interest to ask whether Markov equilibria, as a class, manifest the same patterns highlighted in our main result, Proposition 4. It turns out that they do not. The following two propositions establish both existence and a “uniformity” property; the latter provides additional justification for our focus on history-dependent strategies.

PROPOSITION 6: Define $\beta^* \equiv (1 - \delta)/[\delta(\alpha - 1)]$. Then:

- (i) If $\beta \geq (>)\beta^*$, there exists a linear Markov equilibrium $\phi(A) = kA$ with $k \geq (>)1$.
- (ii) If $\beta < \beta^*$, there exists a Markov equilibrium policy ϕ with $\phi(A) \leq A$ for all $A \geq B$.

This proposition establishes existence of a Markov equilibrium for every value of β . Part (i) also reveals that when $\beta > \beta^*$, there are linear Markov equilibria with strict accumulation. For such β , it follows that the Markov equilibrium set is necessarily uniform, as strong accumulation is a possible equilibrium outcome irrespective of the initial conditions. Part (ii) gets us part of the way toward the same conclusion when $\beta < \beta^*$, in that it establishes the existence of an everywhere-non-accumulating Markov equilibrium. However, it does not rule out the existence of somewhere-accumulating Markov equilibria, which might render the entire set of Markov equilibria non-uniform. Accordingly, we present a second result, which completely resolves the issue of uniformity.

PROPOSITION 7: Suppose there exists a Markov equilibrium ϕ with $\phi(A) > A$ for some $A \geq B$. Then there exists a Markov equilibrium ϕ' with $\phi'(A) \geq A$ for all $A \geq B$, with strict inequality if $\beta \neq \beta^* \equiv (1 - \delta)/[\delta(\alpha - 1)]$.

In short, uniformity is generally a feature of the Markov equilibrium set. Our analysis therefore depends on history-dependence, not only to represent the psychological phenomenon of self-reinforcement, but also to generate our central conclusions concerning the relationship between initial wealth and self-control.

The proof of Proposition 7 proceeds as follows. We have already seen that when β exceeds β^* , the Markov set is uniform. Focusing then on $\beta < \beta^*$, we note that, if there is an equilibrium with $\phi(A) > A$ for some $A > B$, we can create a scaled-down version ϕ' such that $\phi'(B) > B$. It is easy to check that ϕ' must be nondecreasing. Accordingly, if it ever passes below the 45° line, there must be some asset level S at which $\phi'(S) = S$. But then, from S , the individual could decumulate slightly, and count on his future selves to accumulate back to S . With $\beta < \beta^*$, that alternative necessarily yields a higher payoff than choosing S , which contradicts the supposition that ϕ' is an equilibrium. Therefore, the Markov set must also be uniform for $\beta < \beta^*$. See the Supplemental Material for details.

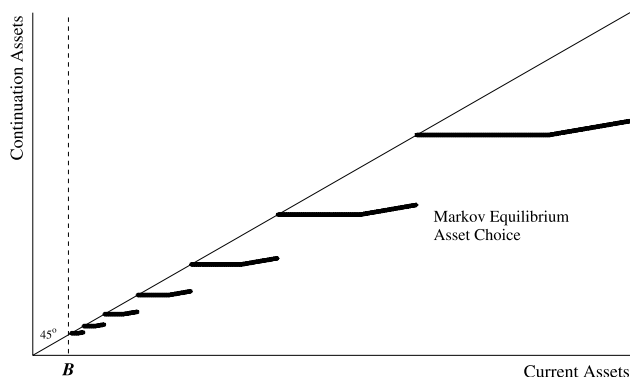


FIGURE 5.—Markov equilibrium with periodic maintenance.

Despite this negative result, it is worth noting that Markov equilibria can give rise to asset traps. Specifically, in a (weakly) decumulating Markov equilibrium,³⁹ there may be levels of A at which asset *maintenance* is possible. Figure 5 shows one such case (which reflects actual numerical examples). To see how such equilibria arise, suppose that β is relatively small, so that starting just above the lower bound B , the individual exhausts liquid assets. As the initial asset level increases, the subsequent (proportional) decline in assets is greater, and hence the (normalized) value of the trajectory is lower. Consequently, as long as β is not too small, there comes a point (call it S) at which asset maintenance yields equivalent value, in which case one can construct a Markov equilibrium in which S is chosen from S . Moreover, for asset levels above S , the game “scales up” by the factor $\mu = S/B$, just as described in Observation 2. Applying this logic recursively, we generate an infinite sequence of asset levels $\{B, S, \mu S, \mu^2 S, \dots\}$ for which maintenance is possible, each of which acts as an asset trap.⁴⁰

³⁹Recall that weakly decumulating Markov equilibria exist for $\beta < \beta^*$.

⁴⁰Yet another alternative is to focus on subgame-perfect equilibria supported by Markov-reversion. Because this alternative involves permanent punishments (in which the individual never “climbs back on the wagon”), it strikes us as unappealing. That said, reversion to a cyclical Markov equilibrium, such as the one exhibited in the text, cannot give rise to the patterns highlighted in Proposition 4. Specifically, it is easy to show that any equilibrium of this type starting from an asset level in $[\mu^k S, \mu^{k+1} S]$ for any $k > 0$ can be scaled down to a starting point in any $[\mu^j S, \mu^{j+1} S]$ with $j < k$. It follows that the equilibrium set supportable by reversion to a cyclical Markov equilibrium is also cyclical. That said, there may also be strictly decumulating Markov equilibria, and our computational analysis has identified cases in which reversion to those equilibria supports the qualitative pattern highlighted in Proposition 4; see the Supplemental Material. However, whether reversion to the *worst* Markov equilibrium generally yields a counterpart to Proposition 4 is an open question.

7. SOME ADDITIONAL IMPLICATIONS

In this section, we explore the broader implications of our analysis for behavior and policy (aside from the benefits of “priming the pump” for those caught in the poverty trap). We touch on four topics: the effect on saving of easier access to credit; the demand for external commitment devices; the design of accounts to promote saving; and the observed variation in marginal propensities to consume from wealth across classes of resource claims.

7.1. *The Effects of Easier Access to Credit*

The decline in saving rates among U.S. households during the latter part of the 20th century is sometimes attributed, at least in part, to institutional developments that progressively improved access to credit.⁴¹ Conventional theory predicts that more abundant (and cheaper) credit reduces aggregate saving. In contrast, our model has more nuanced implications.

Conceptually, comparative statics with respect to the level of the borrowing constraint (B) are straightforward. Although the constraint destroys scale neutrality, a change in B simply rescales the equilibrium set. Thus, we can reinterpret Proposition 4 as showing that there are two values, μ' and μ'' , with $1 < \mu' \leq \mu'' < \infty$, such that a poverty trap exists whenever $A/B < \mu'$, while unbounded accumulation is possible whenever $A/B > \mu''$.

It follows that the effect on saving of relaxing the credit limit depends on the level of initial assets. The direct effect of such a relaxation is to reduce B , for example, from B_1 to $B_2 < B_1$, thereby increasing the ratio A/B for everyone. That change may allow an individual to escape the poverty trap (i.e., if $A/B_1 < \mu' < A/B_2$), and may even enable him to accumulate assets indefinitely (i.e., if, in addition, $A/B_2 > \mu''$). However, there is also a downside to easy credit: those whose assets remain below $\mu'B_2$ will slide even further into poverty. In any given context, either the first effect or the second may be more prevalent. Notably, [Karlan and Zinman \(2010\)](#) showed that a field experiment that expanded access to costly consumer credit in South Africa on average improved economic self-sufficiency, intra-household control, community status, and overall optimism. Similarly, [Dobbie and Skiba \(2013\)](#) showed that larger payday loans lead to lower rates of default.

7.2. *The Demand for Commitment Devices*

As noted in Section 1, a demand for precommitment has been documented for poor households in developing countries. However, there is surprisingly

⁴¹See, for example, [Bacchetta and Gerlach \(1997\)](#), [Ludvigson \(1999\)](#), [Parker \(2000\)](#), and [Glick and Lansing \(2011\)](#).

little evidence that this demand is more widespread,⁴² and as a result nagging doubts about the importance of (sophisticated) time inconsistency persist. Skeptics wonder why, if time inconsistency is so prevalent, the market provides few commitment devices, and why unambiguous examples in the field are so difficult to find.

Our analysis provides a potential resolution to this puzzle. Because full-precommitment is neither possible nor desirable (due to the value of flexibility), people must rely to some extent on internal mechanisms for self-control. Significantly, the use of external commitments may undermine the efficacy of those internal mechanisms by rendering ineffective the individual's personal rules. As an illustration, consider an external commitment that "locks up" certain assets in an illiquid savings account. The direct effect of that commitment is to increase B , the lower bound on net worth, say from B_1 to $B_2 > B_1$. The impact on saving is then the same as for a tightening of the credit constraint. In particular, defining μ' and μ'' as above, if $A/B_1 > \mu' > A/B_2$, the external commitment would render unbounded accumulation infeasible, and if $\mu' > A/B_2$, it would induce the individual to deplete all of her other (liquid) assets. Accordingly, people may have powerful reasons to avoid (partial) external commitments.

In our model, the people who value external commitments are those who are asset-poor relative to their credit limits. The asset-rich would rather save on their own. By the same reasoning, if we assume B is a constant fraction of permanent income, the *income-rich* would exhibit a desire for external commitment, while the *income-poor* would prefer to rely on internal mechanisms. To be sure, the *income-rich* may also be asset-rich, so that the net effect is ambiguous. Nevertheless, the theory yields empirical predictions that are, in principle, testable.

7.3. *Designing Accounts to Promote Saving*

Some policies encourage saving by providing special accounts for specific purposes, such as retirement, education, or medical expenses. Virtually all such accounts entail commitments, the nature of which differs considerably across programs. For example, the degree to which savings are "locked up" until retirement varies across pension programs. For Social Security and many private plans (especially of the defined benefit variety), lock-up is absolute. IRAs impose a moderate early withdrawal penalty of 10%. For 401(k)s and 403(b)s, the

⁴²Studies documenting a demand for precommitment in *developed* countries are scarce. Exceptions include Ariely and Wertenbroch (2002) on homework assignments, Beshears, Choi, Laibson, and Madrian (2011) on commitment savings devices in the United States, and Houser, Schunk, Winter, and Xiao (2010) on a laboratory experiment in which subjects gain relevant experience. Gine, Karlan, and Zinman (2010) wrote that "there is little field evidence on the demand for or effectiveness of such commitment devices." For recent surveys, see Bryan, Karlan, and Nelson (2010) and DellaVigna (2009).

same 10% penalty applies, but employers can also impose additional restrictions and, as an example, often limit early withdrawals to funds contributed by the employee. After retirement, the lock-up continues in a modified form for Social Security and many private plans: income is paid out at a specified rate, or investment in annuities is mandated. In contrast, IRAs and many other private plans effectively unlock the funds at retirement, making them highly liquid. In addition, participants in retirement savings programs often precommit to contributions. For Social Security and many private plans, contributions are inflexible. For 401(k)s and 403(b)s, they are adjustable, but only with a significant delay (for example, a pay period). Only IRA contributions are fully flexible.

Our analysis potentially sheds light on the ways in which the commitment features of special savings accounts affect saving. Caution is warranted, inasmuch as our model lacks a retirement period, and therefore maps imperfectly to a realistic life-cycle planning problem. Still, one can interpret it as providing a stylized representation of saving decisions during the accumulation phase of the life cycle.

An asset lock-in has both an upside and a downside. The upside is that it can compensate for the absence of self-control when assets are low; the downside is that it can undermine internal self-control mechanisms when assets are high. Because these effects materialize at different asset levels, it is in principle possible to design programs that capitalize on the upside while avoiding the downside, for example by locking up all funds until some personally chosen asset target is achieved, and then removing the lock (irreversibly), making all funds liquid. Pilot programs with such features have indeed been tested in developing countries.⁴³

Formalizing the preceding intuition is not entirely straightforward. In our simple model, lock-up would prevent people with low assets from decumulating, but it would not necessarily enable them to employ personal rules that support contributions to the account in the first place (and might undermine that ability). Furthermore, there is an obviously superior policy alternative that achieves the Ramsey outcome: require participants to select contributions and withdrawals one period in advance.

Our intuition concerning account design is nevertheless borne out in a slightly more elaborate model that incorporates preference shocks (for example, those reflecting transient needs associated with illnesses requiring costly medical care). In such cases, an exclusive reliance on external commitment is problematic. Suppose in particular that flow utility is given by

$$u(c, \eta) = \eta \frac{c^{1-\sigma}}{1-\sigma},$$

⁴³See Ashraf, Karlan, and Yin (2006), as well as Karlan et al. (2010).

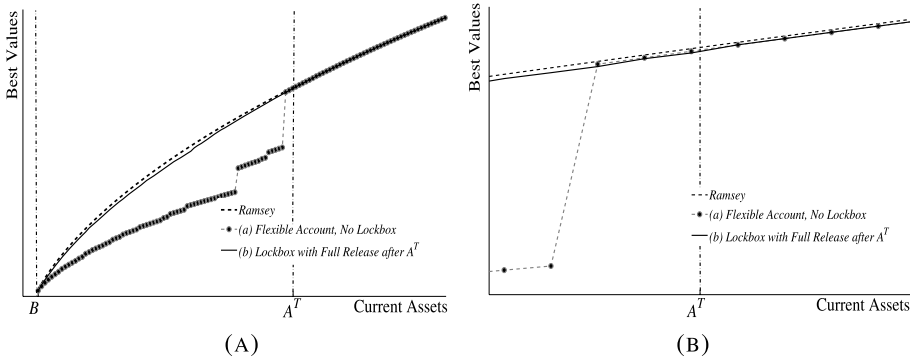


FIGURE 6.—Equilibrium values: lockbox with unlocking. (A) Overall. (B) Zoom.

where η is an independent and identically distributed (i.i.d.) random variable realized at the outset of each period. If the distribution of η encompasses sufficiently low values, the individual will contribute to a lock-up account in some states of nature even when assets are low. Moreover, committing to contributions one period in advance sacrifices the individual’s ability to condition consumption on the realization of η , and consequently does not deliver the generalized Ramsey solution.

Due to the complexity of the extended model, we analyze it computationally; see the Supplemental Material for details. Numerical solutions generally confirm our intuition. Figures 6 and 7 depict results for an illustrative case.⁴⁴ Figure 6, panel A, shows the highest achievable equilibrium value as a function

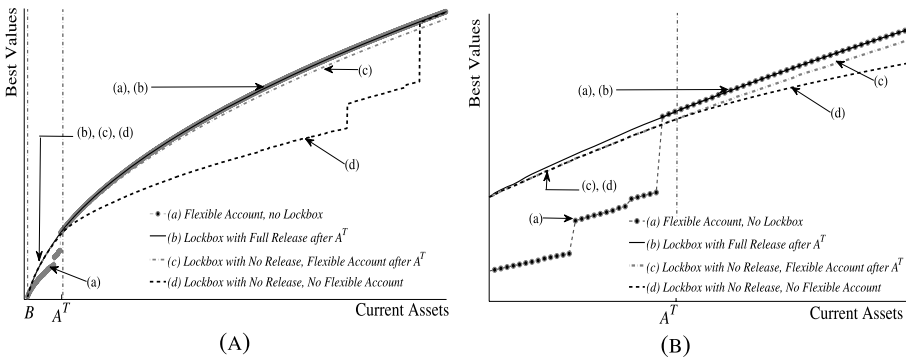


FIGURE 7.—Alternative lockbox regimes. (A) Overall. (B) Zoom.

⁴⁴For the parameters, we take $A = 1.3$, $\sigma = 0.5$, $\delta = 0.8$, and $\beta = 0.4$. The taste shock η takes two values, 0.8 (with probability 0.3) and 1.1 (with probability 0.7). The horizontal axis starts at $B = 0.5$, and v is taken to be a tiny number.

of initial assets for two policy regimes: in the first, labeled (a), only a standard savings account is available; in the second, labeled (b), the individual has access to an account that locks up principal but unlocks once an appropriate target is reached. For policy (a), the highest value jumps upward when the individual has sufficient assets to save on her own. We have chosen the lockbox target (labeled A^T) to be slightly higher than the jump point: with a lower target, she would deplete her assets once the account is unlocked.

At “low” asset levels below the jump point, the individual fares better with the lock-up account than with the standard one. For asset levels above the lockbox target, the two curves must coincide (because the account is unlocked). Notice that the lock-up account allows the individual to achieve values close to the Ramsey solution (also shown). In our example, it does not quite reach that ideal; see the magnified value functions around the asset target in panel B.

Critically, *both* the lock-up *and* the subsequent release are important. Figure 7 replicates the highest equilibrium value functions for the standard and lock-up accounts, and adds lines for two additional policy regimes. In both, principal in the special account remains locked up forever, there is no release, but interest is always accessible. Before the asset target is reached, all savings are channeled into the lockbox and added to the existing principal. With policy (d), there is a finite asset target, after which additional savings are not put into the lockbox, but instead stored in a fully flexible, standard account with access to principal and interest, while the principal in the lockbox account continues to remain inaccessible. With policy (c), a flexible account is not accessible at any asset level and all savings are treated as contributions to the lockbox account.⁴⁵ Figure 7 shows that policy (c) reduces equilibrium value relative to policy (b) (and panel B magnifies the region around the jump point for clarity); though it promotes saving, it does not allow the individual’s self-control mechanisms to take over. She fares even worse with policy (d); though it helps her achieve the target A^T , its subsequent effect is to scale up the credit constraint from B to A^T , thereby creating a new “poverty trap” at that level. In short, though a permanent lockbox is helpful, only one that is eventually dismantled, such as policy (b), can reap the benefits of enabling effective personal rules.

7.4. *Asset-Specific Marginal Propensities to Consume*

It has been observed that marginal propensities to consume might differ across classes of resource claims (e.g., between income flows and liquid assets); see Hatsopoulos, Krugman, and Poterba (1989), Thaler (1990), and Laibson (1997). Our model provides a potential explanation. Recall from Section 2.1 that the lower bound on assets (B) may reflect limitations on the ability to borrow against future earned income, so that, for example, $B = \lambda \frac{\alpha}{\alpha-1} y$ for some

⁴⁵Because it is easier to accumulate assets in a lockbox account than in a standard account, the lack of access to a standard account is not consequential.

$\lambda \in (0, 1)$. An increase in financial assets then leaves B unchanged and increases A_t/B , potentially enhancing self-control according to our main result. As a consequence, the marginal propensity to consume out of an unforeseen change in financial assets could be quite low. In contrast, an equivalent increase in human capital, $\frac{\alpha}{\alpha-1}y$, leaves B/y unchanged, so A_t/B falls, potentially undermining self-control. Therefore, the marginal propensity to consume from an unforeseen change in permanent income could be quite high. More generally, as long as B is an *increasing* function of permanent income, the marginal propensity to consume will tend to be higher for permanent income than for liquid assets. This provides a new perspective on the “excess sensitivity” of consumption to income.

8. CONCLUSION

If people fundamentally differ in their capacities for exercising self-control, then the impulsive ones are more likely to deplete their assets. Yet as we emphasize, there may also be a strong feedback effect from indigence to poor self-control. Our main result shows that poverty can undermine the ability to exercise self-control through contingent self-reinforcement, while wealth can enhance that ability. While there are many other explanations for the persistence of poverty, the endogenous failure of self-control emerges as a potentially important contributory factor.

We envisage further progress along three distinct but complementary lines of research. First, in the spirit of [Bernheim, Ray, and Yeltekin \(1999\)](#), there is a need for well-specified models that incorporate realistic features of the economic environment while remaining amenable to computational solutions. We have provided some preliminary analysis of this type in Section 7.3, but much more work remains, especially if the objective is to develop a deeper understanding of mechanism design in the context of commitment problems. For instance, while our analysis points to some intriguing relationships between external commitment devices (such as fixed deposit or lockbox retirement accounts) and the efficacy of contingent self-reinforcement, a thorough analysis of that topic is beyond the scope of the present paper. Computation is useful because it can inform analytical questions that are currently beyond our theoretical expertise. Indeed, even in the current paper, we have relied on computations to verify that the equilibrium set is generally non-uniform for some intermediate range of β , and that the two thresholds identified in Proposition 4 generally coincide.

That takes us to the second line, which concerns theory. We have adopted the point of view that an agent with self-control problems is to be modeled as a strategically sophisticated actor who tempers her own indiscretions with the precise use of personal, history-dependent strategies. Part of the discomfort one might feel with this approach stems from a tension between the assumed time-inconsistent impulsiveness and the exact computations that dynamic strategic reasoning entails. One might seek to alleviate this complexity

in one of three ways: (i) assume that consumers reason out their choices by thinking through analogous but simpler and hence more tractable problems (as suggested in Section 5.4); (ii) consider other solution concepts that could make for easier computation, such as ϵ -equilibrium (see, e.g., Radner (1980)); or (iii) seek other more tractable descriptions of present bias. With respect to (iii), one interesting possibility would posit that consumers confine attention to a limited horizon that rolls forward as time passes; see, for example, Arrow (1973), Kohlberg (1976), and Jehiel (1995, 2001). To be sure, in this case one would need a theory that explains (rather than assumes) why the horizons of the poor are more limited than those of the rich.

The third line of research concerns empirical analysis. We have referred to a number of empirical studies, but our investigations yield new predictions that are worth testing. Perhaps most striking is the central prediction that wealth effects (on self-control) are consequences of imperfect credit markets. This is to be contrasted with the alternative hypothesis (Banerjee and Mullainathan (2010)) that such wealth effects stem from preferences—specifically, the assumption that the poor are more susceptible to temptation. It would also be interesting to investigate the empirical interrelationship between external and internal modes of self-control, extending the analysis in Section 7.3.

At the broadest level, this paper is a contribution to the behavioral economics of poverty. Self-control is just one of several pertinent behavioral considerations; others include internally and socially generated aspirations, the reliance on role models, decisions to acquire detailed knowledge about rates of return from investments in health and education, and other types of informational and psychological distortions that are traceable to the conditions of poverty. Which of these considerations tend to amplify initial conditions, and what types of interventions will promote convergence and growth by nullifying those conditions? Progress toward answering those questions—both theoretically and empirically—would be of immense significance.

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