Journal of International Economics 13 (1982) 105–134. North-Holland Publishing Company

FEASIBLE ALTERNATIVES UNDER DETERIORATING TERMS OF TRADE*

Tapan MITRA, Mukul MAJUMDAR and Debraj RAY

Cornell University, Ithaca, NY 14853, USA

Received January 1981, revised version received June 1981

The paper presents a dynamic general equilibrium model of a small open economy which employs an essential imported input in production. The economy is said to be capable of survival, if, given the technology and the time-path of world prices, there is at least one sequence of feasible decisions on intertemporal resource allocation sustaining a positive stationary level of consumption indefinitely. The principal results of the paper characterize necessary and sufficient conditions on the technology and the rate of decline of the terms of trade that ensure survival.

1. Introduction

The problems of economic growth under a progressively tightening balance of payments constraint have been studied fairly extensively, at least in the literature on the economics of development. Allied work has been carried out directly attacking the question of a worsening trade deficit, leading to the various studies on foreign aid and the numerous debates on import-substitution and export promotion as alternative methods of tackling a deteriorating trade situation.[1]

The recent energy crisis, brought on by the steep rise in oil prices, now poses a similar threat to much of the developed world. In addition, an import substitution drive, such as the development of an alternative energy source, carries with it none of the usual companions: the 'infant industry' and protection arguments. Import substitution is of a once-and-for-all nature: once an adequate alternative source is found, the problem is solved, but there is no guarantee of its discovery. In fact, for countries such as Japan, a domestic source of oil just does not exist. Given the bounds on foreign aid,

*The research reported here was supported by the National Science Foundation. Mukul Majumdar wishes to thank Dean Alain Seznec for approving of arrangements that facilitated completion of the present manuscript.

[1]A sample of the kind of work done in this area could include Chenery and Bruno (1962), Singer (1950), and Prebisch (1959). For studies in foreign aid Bhagwati and Eckaus (1970) is a representative starting point, and a number of papers in the Economic and Political Weekly debate the import substitution versus export promotion issue (see the issues in the early seventies).

we are left with the possibilities of export promotion and technical progress, as tools to design an 'optimal' policy with.

Given the essentially dynamic nature of the problem, it is important to see whether the traditional analysis of optimal accumulation or intertemporal resource allocation provides a useful reference point. This literature usually starts with a precise description of the set of all feasible plans (given the technology, the supply of labor, etc.), and goes on to consider optimality criteria for evaluating alternative plans. In many of the well-known models,[2] the feasible set is typically rich enough to allow for significant expansions of production and consumption possibilities, so that attention is largely focussed on the merits of alternative choice criteria, and the qualitative properties of optimal programs under such criteria.

The recent re-examination of intertemporal allocation theory with an explicit recognition of the role of exhaustible resources[3] as essential inputs in production has, however, raised a class of questions pertaining to the structure of the feasible set itself.[4] In particular, the richness of the feasible set is examined by determining whether or not it is possible for the economy to maintain a positive stationary flow of consumption *indefinitely* over time. The fact that an economy employs a nonproducible good as an essential input is not a reason for concern, if through capital accumulation and appropriate substitution between producible and nonproducible inputs it is possible to sustain an adequate level of consumption in all periods. On the other hand, if for *every* feasible program it is impossible to maintain a positive stationary consumption level over all time periods, the problem of survival, rather than of efficient or optimal choice, becomes uncomfortably relevant.

It is with this possibility that we are concerned with in our analysis of a 'small' open economy, which employs an essential imported input in production. The economy produces two goods representing 'industrial output' and 'agricultural output'. The industrial product may be exported or invested; if invested, it augments the capital stock of the economy. We assume that such investment is irreversible; in other words, the capital stock itself cannot be consumed. The capital stock may be allocated in any way between the two sectors for use as an input in production. The agricultural output may be exported, but is not invested.

The economy uses an 'intermediate' input which is imported (a convenient interpretation is that it is an 'energy input'), and the price of this good is

[2]We have in mind the impressive literature based on the contributions of Ramsey (1928), Von Neumann (1945/6), and Malinvaud (1969).

[3]See Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974), Cass and Mitra (1979), among others.

[4]However, the models seldom consider both capital accumulation and trading with other economies simultaneously.

determined exogenously, as are the prices of all exports and imports. This is, of course, the standard 'small economy' assumption of international trade theory. Further, it is required that the economy must be in a balance-of-trade equilibrium, in the sense that the value of exports must equal the value of imports in any period.

The economy is initially endowed with a capital stock, and faces a given set of (international) prices. The capital stock is divided among the industrial and agricultural sectors, and a certain quantity of 'energy input' imported, which is allocated between the two sectors. Agricultural and industrial outputs are then produced, and parts of these are exported to pay for the imports at the beginning of the period. A fraction of the industrial output may be invested; and all remaining output is consumed. In the next period, the economy is equipped with a capital stock equal to the old capital stock plus investment made in the earlier period, and faces a fresh set of prices. The entire process then repeats itself.

The economy is said to be capable of *survival*, if, given the sequence of world prices and the technology, there is at least one feasible sequence of decisions sutaining some positive stationary level of consumption of both goods indefinitely.

The principal result of this paper is the presentation of necessary and sufficient conditions on the technology and terms of trade, ensuring survival. Theorems 2.1 and 2.2 taken together provide an almost-complete characterization of survival.

There are dangers in adopting a purely partial equilibrium approach to the situation. A typical 'partial equilibrium' reaction to the problems created by rising input prices is to stress the possibilities of promoting exports, but it is intuitive that export promotion can often be achieved only by reducing current consumption, or by curtailing investment which will in turn ensure a loss of future consumption. This is brought out clearly in the simplified general equilibrium framework that we set up.

It is equally intuitive that the survival problem is linked to the dynamic behaviour of the terms of trade (given the 'small country' assumption); however, a precise characterization of price behaviour and technology does require a careful examination of the structure of our model. It turns out that an economy might be capable of survival even with a continued indefinite, unbounded decline in the terms of trade, provided that the deterioration does *not* occur at *too* fast a rate (see the example in section 4).

In the context of an autarkic Cobb–Douglas economy endowed with a positive stock of a nonproducible resource used as an essential input, Solow (1974) pointed out that indefinite maintenance of a positive stationary level of consumption is possible if the share of capital in total ouput exceeds that of the resource. Appropriate conditions for a Cobb–Douglas economy in which technical progress and population growth occur at exogenously specified exponential rates were studied by Stiglitz (1974). It is of interest to note that a trading economy may well be capable of survival even though the production function fails to satisfy the Solow condition.[5] On the other hand, there exist situations where survival is impossible, yet possessing the property that total resource imports may be infinite. This points to the crucial role played by the terms of trade path itself, independent of the total quantities of resource imported.

In a well-known survey, Bhagwati (1966) argued that *the* central limitation of the international trade literature was its inability to handle intermediate products and capital goods which figure prominently in world trade. He also noted that 'dynamic propositions in international trade are comparatively few, and bear no trace of any uniform design'. It is hoped that the results here add to the list of 'comparatively few' propositions. The model, spelled out in detail in section 2, is a particularly simple general equilibrium system, and captures the intertemporal allocation problem with an explicit capital accumulation process. Thus, it differs from the literature building on the Hicksian approach (1953) which is essentially comparative static in nature, and the more recent models of Vousden (1974) and Kemp–Suzuki (1975) which omit capital as a factor of production. To our knowledge, the literature on trade with imported inputs, or on dynamic models analyzing growth and patterns of trade over time, does not exhibit any result any result along the lines adopted here.

In section 3, we remove some simplifying assumptions, and indicate how the analysis may be extended in order to incorporate technical progress. In section 4, the implications of the more 'abstract' model of sections 2 and 3 are clarified by means of a specific example using Cobb–Douglas production functions, and we analyze the possibilities of storing the imported input, along with the implications of such storage.

We do acknowledge that the study of one country in isolation, and the retention of the 'small economy' assumption over time, precludes a serious examination of feedback effects, especially in the determination of resource prices. A better understanding of growth and trade patterns would require an explicit treatment of at least two trading countries or two blocks, with a recognition of the game theoretic elements involved. How a more general, yet tractable dynamic formulation may be achieved is not yet clear to us. We hope to study this problem in the future.

2. The model

To recapitulate, we consider an economy producing 'agricultural' and 'industrial' goods with the aid of a durable 'capital' good and an imported

[5]To create a comparable setting, interpret the imported input as flows from a foreign resource pool.

'resource' ('energy'). Both the inputs are indispensable in production, and consumption of both goods is essential for 'survival'. The industrial good can be consumed, exported or invested. Investment, which is irreversible, augments the stock of (perfectly shiftable) capital. The agricultural good can be either consumed or exported.

A few simplifications, mostly for expository reasons, are now discussed and clarified before the formal model is set up.

(1) *The economy is not endowed with a domestic stock of the resource, and is therefore forced to import it.* This is a reasonable approximation for all economies with resource needs far exceeding availability, and enables us to focus on the trading aspects of the problem. However, in the context of an example using Cobb–Douglas production functions (section 4), we indicate how this assumption may be relaxed and the basic result on the implications of rapidly deteriorating terms of trade extended.

(2) *The resource is not directly consumed.* This assumption is only for expository simplicity. Indispensability in consumption will enhance the strength of our findings.

(3) *The scarce resource alone is imported.* This, too, is an assumption for ease of exposition. The analysis could be directly extended to allow for the imports of agricultural products. The qualitative nature of the conclusions will be unaffected, although at several steps one needs to examine the long-run behaviour of the relative prices of such imported commodities.

(4) *The economy is 'small' (and remains so over time).* In other words, the prices of exports and imports are determined by the 'world market', and domestic policy makers take these as given parameters. This simplification is drastic, and few, if any, countries would conform to it exactly, but it aids in isolating the bare essentials of the problem. As an illustration, one concedes that countries with significant political power may also possess significant market power and effective bargaining skills, but the recent experience with energy prices seems to indicate that even major importers of energy face deteriorating terms of trade, and it is precisely the effects of such deterioration that we would like to single out. Furthermore, if the economy faces a downward sloping curve for its export and in the extreme case has a monopoly in the export market, the techniques of proof are directly applicable, with no change in the qualitative results. Similarly, if a large economy serves to pull up prices with inflated demands, the basic conclusions still hold.

(5) *The balance of payments is in equilibrium in all periods.* This rules out the possibility of studying the effects of foreign borrowing and trade. However, at the cost of more involved computations, it should be

possible to introduce various forms of foreign aid, and to obtain results similar to theorems 2.1 and 2.2 if 'reasonable bounds' on such aid are imposed.

(6) *The resource is not durable (or storable).* We shall first make the assumption that the resource is not storable, so that the entire amount of import must be used up as an input in the production process. It is surely not always realistic to assume costless storage, and the example in section 4 indicates that if the resource is durable, but that either the resource or the capital good is subject to depreciation during storage, one of the major results continues to hold. In any case, the assumption that the resource is nonstorable enables us to draw a useful distinction between two types of problems. One is the question of efficient or optimal allocation of a fixed durable stock of a nonproducible good. This is the central theme of the Solow type studies, and the assumption of costless storage or nondepreciation is perhaps a useful simplification in that context. The other question deals with the effects of price movements *per se* when an economy crucially relies on a good that is not producible, but available through trading with the outside world. This is the focus of our analysis, and nonstorability helps us in analyzing the terms-of-trade effects in isolation.

It seems desirable to amplify the implications of the results that we present below, before confusion occurs as to exactly what we are stressing. The extreme sharpness of the conclusion is surely due to the simplified structure of the model. We do not claim that exploding oil prices will literally wipe out a small economy relying on imported oil as an essential input. What we do assert is that substantial, *qualitative* transformation of the production process is essential for a country facing the dismal prospect of a continued, rapid deterioration in its terms of trade. To the extent that human beings had survived before oil-intensive production techniques were developed, survival is guaranteed, regardless of what happens to oil prices. However, the 'normal' way of life (or the 'acceptable' standard of living) will surely be endangered in an economy which fails to make appropriate adjustments, and continues to rely on imported oil as a crucial input. For example, an oil-based transport system may collapse while still preserving the possibility of survival in a harsh biological sense, but no one would hesitate to regard such a collapse as a situation of overwhelming economic decay.

Possible extensions of our analysis to consider exogenously postulated minimal levels of consumption or growth rates would certainly be of interest. However, in the absence of technical progress, one may end up with only 'negative' results, as will be clear once we state and discuss our propositions formally.

2.1. The survival problem in an economy without technical change

2.1.1. Notation

In what follows, the subscript t refers to the time period. Moreover, the following symbols are used:

A: agricultural output
X: industrial output
C_i: consumption of output i; $i=A,X$
E_i: export of output i; $i=A,X$
K_i: capital stock in sector i; $i=A,X$
R_i: resource used in sector i; $i=A,X$
K: total capital stock
R: total input of the resource
I: investment
q: price of X relative to A
p: price of R relative to A

Finally, $\mathbf{R}_+$ (resp. $\mathbf{R}^2_+$) denotes the set of all non-negative reals (resp. all pairs of non-negative reals).

2.1.2. The structure of the economy

Two commodities, the agricultural and industrial outputs, are produced, in any period t, by 'capital' (the input of industrial good) and 'resource' (an essential imported input that does not enter into consumption directly), according to production functions satisfying standard assumptions. Thus, we have:

$$A_t = A(K_{At}, R_{At}), \qquad t \geqq 0, \tag{2.1}$$

$$X_t = X(K_{Xt}, R_{Xt}), \qquad t \geqq 0. \tag{2.2}$$

The following assumptions on the technology are made:

A.2.1. (a) *The function* $A(\cdot,\cdot):\mathbf{R}^2_+ \to \mathbf{R}_+$ *and* $X(\cdot,\cdot):\mathbf{R}^2_+ \to \mathbf{R}_+$ *are homogenous of degree one in their arguments.* (b) $A(\cdot,\cdot)$ *and* $X(\cdot,\cdot)$ *are twice continuously differentiable. Moreover,*

$$A(0,\cdot) = A(\cdot,0) = X(0,\cdot) = X(\cdot,0) = 0$$

(both factors are essential in production in both sectors),

$$A_K > 0, \quad A_R > 0, \quad X_K > 0, \quad X_R > 0$$

(positive marginal product of the factors in both sectors),

$$A_{KK}<0, \quad A_{RR}<0, \quad X_{KK}<0, \quad X_{RR}<0$$

(diminishing marginal productivity of the factors in both sectors),

$$\lim_{K\to\infty} A_K(\cdot,\cdot)=0, \quad \lim_{K\to 0} A_K(\cdot,\cdot)=\infty,$$

$$\lim_{R\to\infty} A_R(\cdot,\cdot)=0, \quad \lim_{R\to 0} A_R(\cdot,\cdot)=\infty,$$

$$\lim_{K\to\infty} X_K(\cdot,\cdot)=0, \quad \lim_{K\to 0} X_K(\cdot,\cdot)=\infty,$$

$$\lim_{R\to\infty} X_R(\cdot,\cdot)=0, \quad \lim_{R\to 0} X_R(\cdot,\cdot)=\infty$$

(Inada conditions on the boundary behaviour of marginal products).

Given that the agricultural product may be consumed or exported, we have:

$$A_t=C_{At}+E_{At}, \qquad t\geqq 0. \tag{2.3}$$

Given that the industrial product may be consumed, exported or invested, we have:

$$X_t=C_{Xt}+E_{Xt}+I_t, \qquad t\geqq 0. \tag{2.4}$$

Capital, for the moment assumed to be nondepreciating (but see the remarks in section 4), accumulates according to the following rule:

$$K_{t+1}=K_t+I_t, \qquad t\geqq 0. \tag{2.5}$$

Irreversibility of investment is captured by

$$I_t\geqq 0, \qquad t\geqq 0. \tag{2.6}$$

The following relationships describe the intersectoral allocation of inputs:

$$K_t=K_{At}+K_{Xt}, \qquad t\geqq 0, \tag{2.7}$$

$$R_t=R_{At}+R_{Xt}, \qquad t\geqq 0. \tag{2.8}$$

The country is 'small' so that the prices of agricultural and industrial goods as well as the price of the resource are all 'world' prices that the country accepts as given parameters. Thus, the economy faces a price sequence

$\langle q_t, p_t \rangle_{t=0}^{\infty}$.

Finally, the balance of payments is in equilibrium for all t, so that

$$E_{At} + q_t E_{Xt} = p_t R_t, \qquad t \geqq 0. \tag{2.9}$$

The economy is endowed with an initial capital stock K_0.

Let us define a *price path* as any sequence $\langle q_t, p_t \rangle_{t=0}^{\infty}$ such that $q_t > 0$ and $p_t > 0$ for all $t \geqq 0$, and a *consumption program* as any sequence $\langle C_{At}, C_{Xt} \rangle_{t=0}^{\infty}$ with $(C_{At}, C_{Xt}) \in \mathbf{R}_+^2$ for all $t \geqq 0$. In other words, a consumption program is a complete specification of the quantities of the agricultural good and the industrial good that the economy plans to consume in each period.

A consumption program $\langle C_{At}, C_{Xt} \rangle_{t=0}^{\infty}$ is *feasible given a price path* $\langle q_t, p_t \rangle_{t=0}^{\infty}$ *and an initial capital stock* K_0 if there exists a non-negative sequence $\langle X_t, A_t, R_t, R_{At}, R_{Xt}, K_t, K_{At}, K_{Xt}, I_t, E_{At}, E_{Xt} \rangle_{t=0}^{\infty}$ which together with $\langle C_{At}, C_{Xt} \rangle_{t=0}^{\infty}$ satisfy (2.1) through (2.9), given $\langle q_t, p_t \rangle_{t=0}^{\infty}$ and K_0.

Thus, a consumption program is feasible, given $\langle q_t, p_t \rangle$ and initial stock K_0, if it is generated by a plan of investment, intersectoral input allocation, output levels, and exports and imports, consistent with the model described above. Roughly speaking, any feasible consumption program is attainable by means of an appropriate policy of resource allocation and trading decisions. Note that feasibility, as defined here for the economy as a whole, does *not* take as a constraint the nature of the economic system. For example, some programs of allocation and exchange would not be attainable under a capitalist system, given profit maximization and other behavioural assumptions usually associated with such a system. Feasibility is defined here in a broad sense, constrained only by the technology, initial endowments, and world prices.

An economy is said to be *capable of survival* if there is a feasible consumption program with $\inf_{t \geqq 0} C_{Xt} > 0$ and $\inf_{t \geqq 0} C_{At} > 0$. In other words, an economy is capable of survival if there is a feasible program guaranteeing a minimal amount of consumption for both the commodities.

Before stating the main theorems, we introduce some additional notations. In what follows, we set

$$\begin{aligned}
&a(z) \equiv A(1, z), \\
&x(z) = X(1, z), \\
&f(z) \equiv a(z)/z, \qquad z > 0, \\
&g(z) \equiv x(z)/z, \qquad z > 0, \\
&r \equiv R/K, \qquad K > 0.
\end{aligned} \tag{2.10}$$

Theorem 2.1. Suppose that there exists a feasible consumption program with (a) $\inf_{t\geqq 0} C_{At}>0$ *and* (b) $\inf_{t\geqq 0} C_{Xt}>0$. *Then*

$$\sup_{t\geqq 0}\frac{[1/a(Q_t)]}{\prod_{s=0}^{t-1}[1+x(Q_s)]}<\infty, \tag{A}$$

$$\sup_{t\geqq 0}\frac{[1/x(Q_t)]}{\prod_{s=0}^{t-1}[1+x(Q_s)]}<\infty, \tag{X}$$

where

$$Q_t=\max\,[f^{-1}(p_t/2);\, g^{-1}(p_t/2q_t)]. \tag{2.11}$$

We now state a partial converse to the above theorem. To this effect, let us define $Q_t'=\max\,[f^{-1}(\lambda p_t/2);\, g^{-1}(\lambda p_t/2q_t)]$, given some $\lambda>0$.

Theorem 2.2. Suppose that $\langle q_t, p_t\rangle$ *and* $A(\cdot,\cdot)$ *and* $X(\cdot,\cdot)$ *satisfy the following conditions:*

$$\sup_{t\geqq 0}\frac{[1/a(Q_t)]}{\prod_{s=0}^{t-1}[1+Bx(Q_s)]}<\infty,\quad \textit{for all } \lambda>0, \textit{ and } B\in(0,1], \tag{A^*}$$

$$\sup_{t\geqq 0}\frac{[1/x(Q_t)]}{\prod_{s=0}^{t-1}[1+Bx(Q_s)]}<\infty,\quad \textit{for all } \lambda>0, \textit{ and } B\in(0,1]. \tag{X^*}$$

Then there exists a feasible consumption program with (a) $\inf_{t\geqq 0} C_{At}>0$ *and* (b) $\inf_{t\geqq 0} C_{Xt}>0$.

For proofs of these theorems, see the Appendix.

It is not easy to extract much intuition from the theorems in their present state, so we proceed to a special case to bring out the basic idea: the dependence of survival possibilities on the movement of the terms of trade. Since no specific restrictions have been imposed on the nature of price movements or the functional form of the technology, it is perhaps not surprising that the pair of conditions (A) and (X) (or (A^*) and (X^*)) is not entirely transparent. In section 4, the problem is made clearer with the use of an example involving Cobb–Douglas technologies, and other specific implications tackled. However, a few observations are possible even within this more general framework.

First, it is perhaps of some interest to note that neither the arguments nor the concepts employ any criterion of social optimality,[6] especially the well known but rather dubious community indifference curves, which can, strictly speaking, be justified only in exceptional circumstances. The emphasis is primarily on the feasibility issue, and the paper demonstrates the possibility of the feasible set lacking the necessary 'richness' for making any interesting optimality calculations. This will be discussed below.

We leave it as an exercise to show that if $\langle p_t \rangle$ or $\langle p_t/q_t \rangle$ is constant over time (or bounded above), the economy *is* capable of survival, i.e. that (A^*) and (X^*) of theorem 2.2 are satisfied.

The pair of conditions in theorem 2.1 (resp. 2.2) have been labelled (A), (X) (resp. (A^*), (X^*)) to emphasize the fact that if the question of survival was posed entirely in terms of, say, the agricultural good, then conditions (X) and (X^*) have no role to play in the answer to such a question. An identical observation holds for the survival question phrased only in terms of the industrial good. Thus, for example, a necessary condition for $\inf_{t \geqq 0} C_{At} > 0$ is simply condition (A) of theorem 2.1. The theorems in their present form provide, as a result, a useful separation between the characterizations of 'industrial survival' and 'agricultural survival'.

Finally, let us consider an important special case in some detail. Assume that $\langle q_t \rangle$ is a constant sequence, and interpret the imported input as an exhaustible resource (in the exporting country), the price of which is set by a monopolist who follows a Hotelling-type calculation.[7] This motivates a resource price path of the form $p_t = p_0(1+g)^t$, for some positive g. We state the implications for survival in the form of a theorem.

Theorem 2.3. Suppose that $\langle q_t \rangle$ *and* $\langle p_t/q_t \rangle$ *are of the form* $p_t = p_0(1+g)^t$ *and* $(p_t/q_t) = (p_0/q_0)(1+g')^t$ *respectively, where* $g > 0$, $g' > 0$, *and that the industrial production function* $X(\cdot, \cdot)$ *has the property*

$$\liminf_{z \to 0} \frac{zx'(z)}{x(z)} > 0. \tag{2.12}$$

Then there is no feasible consumption program with $\inf_{t \geqq 0} C_{At} > 0$ *and* $\inf_{t \geqq 0} C_{Xt} > 0$.

For a proof, see the Appendix.

Before dwelling on the implications of this somewhat startling result, note that the added requirement on the industrial technology simply amounts to an observation that the resource must be essential in production, not only

[6]Except perhaps the Rawlsian theory of justice by implication. See Solow (1974) for a discussion of this point.

[7]See the literature on exhaustible resources following Hotelling (931).

in the sense captured by A.2.1(b), but also at 'very low' levels of resource use, in terms of some positive output response to a change in resource employment. This property is satisfied trivially by the Cobb–Douglas production function, and other functional forms.[8]

To return to the implications of theorem 2.3: we have an economy with its technology exhibiting substitution properties between the factors of production so that there is always room for a move to a higher capital resource ratio.[9] Yet the results imply that if confronted with an exponential rate of deterioration in its terms of trade, the economy will be unable to meet the balance-of-payments equilibrium constraint *and* the minimal need for maintaining *some* positive level of consumption indefinitely. Note that our definition of a resource allocation program encompasses *all* technologically feasible plans regarding consumption, investment, exports and imports — including *all* conceivable export-promotion strategies. The 'general equilibrium' framework adopted here reiterates the possibility that, after a point, it may be possible to increase exports only at the cost of tightening domestic belts (to drastic limits, as this exercise suggests). It should perhaps be stressed again that we are referring to quantities of consumption goods, and thus the pessimistic message is in no way due to, say, an arbitrary choice of a social welfare function.

3. Technical progress

It is clear from the analysis in the preceding pages that some form of technical progress seems desirable, and in some cases necessary, in the face of rising resource prices. The need for technical progress becomes all the more pronounced when it is recognized that one of the aims of any economy is not only to survive but also to grow. It remains true, of course, that an economy incapable of survival will have nondecreasing consumption programs excluded perforce from its feasible set, and so we continue to use the concept of survival to build up the analysis of technical progress.[10] It is obvious that any reasonable definition of technical progress, once applied, will increase the 'likelihood' of survival, in the sense that a larger collection of price paths will now be compatible with survival. We shall therefore not stop to discuss this aspect of technical progress, but only use the associated theorem as a stepping stone for further analysis.

[8]Less common examples may be cited. For instance, $X(K,R)=A(K+R)^{1/2}K^{1/4}R^{1/4}$; $A>0$; satisfies our conditions A.2.1, with $zx'(z)/x(z)\geq 1/4$ for all $z>0$.

[9]The degree of substitution we are allowing may easily be examined by considering the familiar examples of Cobb–Douglas and CES technologies.

[10]However, as we have already mentioned above, it may be of use to extend the technique employed here to consider the consumption sets containing only nondecreasing programs, or more specific growth paths.

Heuristically speaking, we may conceive of technical progress having two aspects or dimensions: the one works *within* the existing production structure by raising the efficiency of some or all of the inputs in the sense that a larger quantity of output is now obtainable from the same quantities of inputs;[11] the other serves to radically alter the prevailing setup of production — some factors of production are rendered obsolete, while new inputs are introduced. The large-scale introduction of solar energy into an oil-dependent production process would be an example of technical progress of the latter kind.

Though any technical innovation invariably possesses both these dimensions to some extent, analysis using one or the other aspect *alone* as approximating devices seems most fruitful. An examination of the logical issues involved in these two broad aspects within one set of postulates would either lose sight of their qualitative differences, or be general enough to be devoid of all empirical content. The latter aspect outlined above is a fundamentally stochastic one: for example, at no instant of time is the development of an alternative energy source guaranteed, while there are always costs involved in the search for such a source.[12]

First steps in the study of technical progress, approximated by the first aspect alone, could be taken by assuming some deterministic rates of progress in increasing the efficiencies of capital goods and/or scarce resources. We shall pursue this line of inquiry below, while pointing out that analysis of the structure-transforming dimension inherent in technical progress is also of crucial importance.

To minimize algebraic manipulations and to obtain sharper results (in section 4), we shall assume that only the agricultural commodity is exported. We have in mind developing economies whose dominant export items are primary products. In fact, we shall comment below on a form of the model dealing with surplus-labor economies.

We shall mainly be concerned with the comparison of two types of technical progress: capital saving and resource saving. The terms will be defined below, as also the criterion for comparison. The rather nonintuitive result, that under some conditions capital saving technical progress may actually be 'better than' resource saving technical progress, will be found to emerge (section 4).

We define *technical progress* as a sequence $\langle \mu_{At}, \mu_{Xt}, \gamma_{At}, \gamma_{Xt} \rangle$ such that, for all t,

$$A_t = A(\mu_{At} K_{At}, \gamma_{At} R_{At}), \qquad t \geqq 0, \tag{3.1}$$

$$X_t = X(\mu_{Xt} K_{Xt}, \gamma_{Xt}, R_{Xt}), \qquad t \geqq 0. \tag{3.2}$$

[11]For a pioneering exercise in the analysis of such technical progress, see Robinson (1938).

[12]An analysis along the lines of Dasgupta, Heal and Majumdar (1977) or Davison (1978) would perhaps be worthwhile.

Furthermore, $\langle \mu_{At}, \mu_{Xt}, \gamma_{At}, \gamma_{Xt} \rangle$ must satisfy

$$\mu_{it} \geqq \mu_{i(t-1)}, \qquad t \geqq 1, \quad i = (A, X), \tag{3.3}$$

$$\gamma_{it} \geqq \gamma_{i(t-1)}, \qquad t \geqq 1, \quad i = (A, X), \tag{3.4}$$

and

$$\mu_{A0} = \mu_{X0} = \gamma_{A0} = \gamma_{X0} = 1. \tag{3.5}$$

(Pure) capital-saving technical progress is the technical progress sequence $\langle \mu_{At}, \mu_{Xt}, 1, 1 \rangle$.

(Pure) resource-saving technical progress is the technical progress sequence $\langle 1, 1, \gamma_{At}, \gamma_{Xt} \rangle$.

For expositional clarity, we shall be concerned, in the main, with capital-saving technical progress where $\mu_{At} = \mu_{Xt}$ $(= \mu_t$, say), and resource-saving technical progress of the form $\gamma_{At} = \gamma_{Xt}$ $(= \gamma_t$, say).

We now restate the necessary condition for survival (the counterpart of theorem 2.1) for an economy characterized by technical progress as defined above, and exporting only the agricultural product. A corresponding sufficient condition also exists, of course, but we do not state it since these theorems, of themselves, provide little more information than the intuitively obvious fact that the class of price paths permitting survival is larger under technical progress. The necessary condition, however, will be used as a criterion for comparing the relative efficacies of the two types of technical progress.[13]

Theorem 3.1. Suppose there exists a feasible consumption program with (a) $\inf_{t \geq 0} C_{At} > 0$ *and* (b) $\inf_{t \geq 0} C_{Xt} > 0$. *Then* $\langle p_t \rangle$ *must satisfy the following conditions:*

$$\sup_{t \geq 0} \frac{[1/\mu_{At} a[f^{-1}(p_t/\gamma_{At})]]}{\prod_{s=0}^{t-1} [1 + \mu_{Xs} x[(\gamma_{Xs}/\gamma_{As})(\mu_{As}/\mu_{Xs}) f^{-1}(p_s/\gamma_{As})]]} < \infty, \tag{A}$$

$$\sup_{t \geq 0} \frac{[1/\mu_{Xt} x[(\gamma_{Xt}/\gamma_{At})(\mu_{At}/\mu_{Xt}) f^{-1}(p_t/\gamma_{At})]]}{\prod_{s=0}^{t-1} [1 + \mu_{Xs} x[(\gamma_{Xs}/\gamma_{As})(\mu_{As}/\mu_{Xs}) f^{-1}(p_s/\gamma_{As})]]} < \infty. \tag{X}$$

The technique of proof is the same as that of theorem 2.1.

[13]Of course, the sufficient condition may be used with equal validity, or necessary and sufficient conditions for the existence of feasible consumption sets permitting growth (see footnote 10) may be employed. It would be interesting to work out the implications of each criterion; homogeneity of the results could then be construed as a sign of robustness of any of these criteria.

3.1. A comparison criterion

We now provide a criterion which will enable us to compare the 'efficiencies' of the two types of 'pure' technical progress presented above. We shall say that pure capital-saving technical progress *dominates* pure resource-saving technical progress, in sector i ($i=A,X$), if condition (i) ($i=A,X$) of theorem 3.1 under any technical progress sequence $\langle 1,1,\lambda_{At},\lambda_{Xt}\rangle$ implies condition (i) ($i=A,X$) of theorem 3.1 under the corresponding technical progress sequence $\langle\lambda_{At},\lambda_{Xt},1,1\rangle$.

A similar definition characterizes the situations in which resource-saving technical progress dominates capital-saving technical progress.

With any given technical progress sequence, there are associated two sets of price paths. Any price path will have to belong to the first set (corresponding to condition A) if there exists a consumption program with C_{At} bounded away from zero; or to the second set (corresponding to condition X) if there exists a consumption program with C_{Xt} bounded away from zero; or to both if the economy is capable of survival. The definition compares 'identical'[14] paths of pure capital saving and pure resource saving technical progress, and concludes that one dominates the other in a particular sector if the one permits a larger set of price paths to satisfy the necessary condition corresponding to that particular sector.

The definition ignores the possibility that identical paths of the two types of technical progress may well have different costs associated with them. For instance, it is possible that resource saving progress may dominate capital saving progress in a certain sector according to this definition, but the former may also be a more expensive venture. In such cases, the costs will have to be compared before a final decision is arrived at. Costs of technical progress may be modelled in this framework by removing, in each period, part of the capital stock to pay for a given amount of technical progress of a given type.[15]

4. The Cobb–Douglas economy: Some applications and extensions

Consider an economy where both the production functions $A(\cdot,\cdot)$ and $X(\cdot,\cdot)$ are Cobb–Douglas. As with other areas of theoretical and empirical economics, the Cobb–Douglas model has been the 'standard' case in the literature on exhaustible resources — in particular, the 'survival' problem for a closed economy studied by Solow (1974) and Stiglitz (1974) was entirely based on such an assumption regarding the technology. This parametric

[14]The same λ_{At}'s and λ_{Xt}'s are applied to either type of technical progress for purposes of comparison.

[15]See, for example, Davison (1978).

specification enables us to go further in analyzing the implications and possible extensions of our models.

As in section 3, we shall concentrate on the case when *only* the agricultural commodity is exported, and assume no technical progress for the moment.

Instead of (2.1) and (2.2), we now have:

$$A_t = K_{At}^{\beta} R_{At}^{1-\beta}, \qquad 0<\beta<1, \tag{4.1}$$

$$X_t = K_{Xt}^{\alpha} R_{Xt}^{1-\alpha}, \qquad 0<\alpha<1, \tag{4.2}$$

and, setting $E_{Xt}=0$ for all t we get, instead of (2.4),

$$X_t = C_{Xt} + I_t. \tag{4.3}$$

We first restate the theorems of section 2 in the following simplified form:

Theorem 4.1. Suppose that there exists a feasible consumption program with (a) $\inf_{t\geqq 0} C_{At}>0$ *and* (b) $\inf_{t\geqq 0} C_{Xt}>0$. *Then*

$$\sup_{t\geqq 0} \frac{p_t^{(1-\beta)/\beta}}{\prod_{s=0}^{t-1} [1+1/p_s^{(1-\alpha)/\beta}]} < \infty, \tag{4.4}$$

$$\sup_{t\geqq 0} \frac{p_t^{(1-\alpha)/\beta}}{\prod_{s=0}^{t-1} [1+1/p_s^{(1-\alpha)/\beta}]} < \infty. \tag{4.5}$$

Theorem 4.2. Suppose that $\langle p_t \rangle$ *and* (α, β) *satisfy*

$$\sup_{t\geqq 0} \frac{p_t^{(1-\beta)/\beta}}{\prod_{s=0}^{t-1} [1+L/p_s^{(1-\alpha)/\beta}]} < \infty, \qquad \textit{for all } L\in(0,1], \tag{4.6}$$

$$\sup_{t\geqq 0} \frac{p_t^{(1-\alpha)/\beta}}{\prod_{s=0}^{t-1} [1+L/p_s^{(1-\alpha)/\beta}]} < \infty, \qquad \textit{for all } L\in(0,1]. \tag{4.7}$$

Then there exists a feasible consumption program with (a) $\inf_{t\geqq 0} C_{At}>0$ *and* (b) $\inf_{t\geqq 0} C_{Xt}>0$.

It is of interest to note that the conditions (4.4) and (4.5), or (4.6) and (4.7), no longer involve the relative price of the industrial product q. Prices are significant insofar as they serve, directly or indirectly, to offset (or augment) the deterioration in the terms of trade.

Note also that theorem 2.3 on the nonsurvival of the economy under exponential resource price paths hold perforce in this situation; in fact, a direct verification along the lines of the proof of theorem 2.3 requires minimal calculation.

An interesting question deals with the existence of resource price paths, increasing *and* unbounded (with respect to the price of the agricultural commodity), which permit survival. The answer is in the affirmative: for example, consider price paths of the form $p_t = (t+2)^k$, $k > 0$. It follows from the necessary conditions of theorem 4.1 that if $k(1-\alpha)/\beta > 1$, the economy will be incapable of survival, whereas if $k(1-\alpha)/\beta < 1$, the sufficient conditions of theorem 4.2 are satisfied and the economy will be capable of survival. (A proof of this is presented in the Appendix.)

There is a somewhat deeper question, attempting to link up this framework with the autarkic exhaustible resources problem. In the closed economy model, Solow (1974) demonstrated the possibility of survival if the 'elasticity of output with respect to reproducible capital exceeds that with respect to exhaustible resources'.[16] In our trading model, the example of the preceding paragraph indicates the possibility of survival even if the Solow condition is violated: this is only a reflection of the familiar 'gains from trade' thesis. However, the analysis of the exponential price path situation, revealed the impossibility of survival, regardless of whether the Solow conditions hold or not. To erect a framework for comparison, suppose that an economy A (our resource-importing economy) is drawing on a pool of resource situated in economy B, abiding by certain 'rules' stipulated in the balance-of-payments equilibrium condition. Is it true, then, that *whenever* economy A fails to survive, its *total* resource imports must of necessity be finite? In other words, it remains to examine whether our problem is really the closed economy exhaustible resources problem in disguise.

The answer is no; the dynamic *pattern* of trade, as dictated by the price-movement of the resource in conjunction with balance-of-payments equilibrium needs, plays the crucial role, so that economy A may end up not surviving while importing an infinite quantity of the resource. We offer an example for the purpose of illustration; a more realistic case incorporating this phenomenon may be easily worked out.

Let $p_t = 2^t$ for t odd, $p_t = 1$ for t even, and $\alpha = \beta = \frac{1}{2}$. Now,

$$\frac{p_t^{(1-\beta)/\beta}}{\prod_{s=0}^{t-1}[1+1/p_s^{(1-\alpha)/\beta}]} = \frac{p_t^{(1-\alpha)/\beta}}{\prod_{s=0}^{t-1}[1+1/p_s^{(1-\alpha)/\beta}]} = \frac{p_t}{\prod_{s=0}^{t-1}[1+1/p_s]}$$

$$\geq \frac{2^t}{3/2^{t/2}} \to \infty, \qquad \text{for } t \text{ odd.}$$

So (4.4) and (4.5) are violated and the economy is incapable of survival.

[16]Solow (1974, p. 34).

However, consider the feasible program given in the Appendix, where

$$R_t = \frac{\lambda K_0 \prod_{s=0}^{t-1} [1 + \{L/p_s^{(1-\alpha)/\beta}\}]}{p_t^{(1/\beta)}} \geqq \frac{\lambda K_0 (1+L)^t}{1}, \quad \text{for } t \text{ even.}$$

So $R_t \to \infty$ for t even, implying (trivially) that $\sum_{t=0}^{\infty} R_t = \infty$.

4.1. Storage

So far we have assumed that the resource is not storable domestically, but it is important to examine the implication of relaxing this assumption of our model. If costless storage is regarded as feasible, this may be modelled by assuming that the imports are 'nondepreciating'. If, in addition, the capital stock is nondepreciating, we are back to an exhaustible resource model where survival is guaranteed as long as the Solow condition is satisfied, regardless of the behaviour of the terms of trade.

However, if *either* the capital stock *or* the resource stock depreciates at some constant rate, it may be important to analyze some of the implications. We shall present a result for the Cobb–Douglas case, and leave the general characterization as an open question.

We retain the simplifying assumption that the agricultural good alone is exported. Denote by S_t the stock of the resource at time t, and the import of resource at time t by $M_t \cdot R_t$ will denote the input of resource at time t.

The balance-of-payments equilibrium condition now reads:

$$p_t M_t = E_{At}. \tag{4.8}$$

Additionally, if the resource stock depreciates at the rate δ, we have

$$S_t = (1-\delta)S_{t-1} + M_t - R_t, \qquad 0 < \delta < 1, \quad S_t \geqq 0 \text{ for all } t.$$

If the capital stock depreciates at the rate θ,

$$K_t = (1-\theta)K_{t-1} + I_t, \qquad 0 < \theta < 1. \tag{4.10}$$

We study the case where the resource price path is of the exponential form.

Theorem 4.3. Suppose that $\langle p_t \rangle$ satisfies $p_t = p_0(1+g)^t$, $g > 0$, and that either the capital stock or the resource stock depreciates at some fixed rate. Then there exists no feasible program with $\inf_{t \geqq 0} C_{At} > 0$ *and* $\inf_{t \geqq 0} C_{Xt} > 0$.

For a proof, see the Appendix.

The result reinforces the pessimistic implications of this paper, demonstrating the impossibility of survival (under an exponential resource price path) even when the input is storable, though with some cost or depreciation. Where storage is a costless phenomenon (and we leave the reader to judge the validity of this assumption), nonsurvival is still a possibility especially when the capital stock is subject to depreciation.

4.2. Technical progress: The Cobb–Douglas case

Given the implication of the discussion above, it seems necessary to examine the phenomenon of technical progress in greater detail. We may simplify theorem 3.1 using the Cobb–Douglas technology.

Consider (pure) capital-saving technical progress at some rate $\langle \lambda_t, \lambda_t, 1, 1\rangle$. The necessary conditions of theorem 3.1 reduce to

$$\sup_{t\geq 0} \frac{[p_t^{(1-\beta)/\beta}/\lambda_t]}{\prod_{s=0}^{t-1} [1+\lambda_s/p_s^{(1-\alpha)/\beta}]} < \infty, \tag{4.11}$$

$$\sup_{t\geq 0} \frac{[p_t^{(1-\alpha)/\beta}/\lambda_t]}{\prod_{s=0}^{t-1} [1+\lambda_s/p_s^{(1-\alpha)/\beta}]} < \infty. \tag{4.12}$$

Consider now, (pure) resource-saving technical progress at some rate $\langle 1, 1, \lambda_t, \lambda_t\rangle$. The necessary conditions of theorem 3.1 then reduce to

$$\sup_{t\geq 0} \frac{[(p_t/\lambda_t)^{(1-\beta)/\beta}]}{\prod_{s=0}^{t-1} [1+(\lambda_s/p_s)^{(1-\alpha)/\beta}]} < \infty, \tag{4.13}$$

$$\sup_{t\geq 0} \frac{(p_t/\lambda_t)^{(1-\alpha)/\beta}]}{\prod_{s=0}^{t-1} [1+(\lambda_s/p_s)^{(1-\alpha)/\beta}]} < \infty. \tag{4.14}$$

It is easy to verify the following:

Lemma 4.1. (a) $p_t^{(1-\beta)/\beta}/\lambda_t \gtreqless (p_t/\lambda_t)^{(1-\beta)/\beta}$ *according as* $\beta \gtreqless \frac{1}{2}$ *for all* t *(except those where* $\lambda_t = 1$*, when equality always holds).* (b) $1+\lambda_s/p_s^{(1-\alpha)/\beta} \gtreqless 1 + (\lambda_s/p_s)^{(1-\alpha)/\beta}$ *according as* $\alpha+\beta \gtreqless 1$ *for all* s *(except those where* $\lambda_s = 1$*, when equality always holds).*

We now provide some sufficient conditions directing the choice of technical progress in the agricultural sector. The theorem below is an immediate consequence of the lemma:

Theorem 4.4. (a) *Suppose* $\beta > \frac{1}{2}$, *and* $\alpha + \beta > 1$. *Then capital-saving technical progress dominates resource-saving technical progress in agriculture.* (b) *Suppose* $\beta < \frac{1}{2}$, *and* $\alpha + \beta < 1$. *Then resource-saving technical progress dominates capital-saving technical progress in agriculture.*

The proposition brings out the somewhat startling implication that it may sometimes be worthwhile to invest in capital-saving technical progress when *resource* prices are rising. In the general equilibrium type of framework we have adopted, the results should come as no surprise, however. For the resource goes into the production of capital, and saving on capital is an indirect saving on the resource. Moreover, to the extent that capital-saving research is less costly than resource-saving research, at least in terms of the uncertainty involved (regarding, for example, the existence of a viable alternative energy source), the likelihood of capital-saving technical progress being 'better' is increased.

5. Summary and possible extensions

We have attempted to characterize the cases where deteriorating terms of trade lead to the possibility of nonsurvival, in the sense that the economy is unable to maintain *any* positive level of consumption indefinitely. In particular, our examination reveals that in the absence of technical change, nonsurvival is imminent under any exponential resource price path. This result is valid even when the resource is storable, but with resource and/or capital stocks subject to depreciation. However, there do exist increasing and unbounded price paths consistent with the existence of a survival consumption program.

The analysis here may be extended to the situation of surplus-labor economies, where it is possible to hire any amount of labor at any wage rate above some biological minimum.[17]

A behavioural assumption must be made regarding the employment of labor, which constrains the feasible set and therefore the choices of the policymaker. Given the existing socioeconomic structure in many of these less-developed countries it is impossible for the policymaker to set employment levels above certain limits while dictating that the institutional wage payment be observed by the private sector. If this were possible, much of the unemployment problem could be solved! We assume that the private

[17]These are economies (such as India) with 'unlimited supplies of labour' [see Lewis (1954)], in the sense that the available supply of labor far exceeds the economic potential for employing such labor.

sector will soak up the available labor as long as the value of the marginal product of labor is greater than or equal to the institutional minimum.

It may be shown, then, that theorems 2.1 and 2.2 continue to hold, but now the conditions must also take into account the movement of the institutional wage rate over time. In particular, we have the intuitively clear but nevertheless unfortunate result that the survival set of price paths may be expanded if the institutional wage declines over time. It is reasonable to infer that the possibilities of growth are thereby enhanced, too. The tradeoff between growth and distribution rears its ugly head again, though in an unusual form. However, it must be remembered that employment will be on the rise if growth is possible, but our model is certainly not suited to handling these finer problems; the peculiar socioeconomic structure of the less-developed countries must be taken into account.

In any case, technical progress seems to be necessary for economies possessing no domestic stock of oil. It is unlikely that foreign aid can keep pace with the price rise, and import substitution, in the sense of search for a domestic source, is fraught with uncertainty, and in fact, for some countries is well nigh impossible. As has been pointed out, export promotion policies may imply the lowering of domestic consumption levels, an extreme situation arising when the price path of oil does not satisfy the necessary condition of theorem 2.1 so that export promotion policies are as good (or bad) as any other program without technical progress.

When technical progress is possible, it is interesting to compare the alternative forms of progress which are available. We have considered a very specific situation where capital-saving and resource-saving technical progress are compared in relation to the agricultural sector. The criterion used for comparison neglects the costs of technical progress but can hopefully be extended to cover this case. In short, it identifies the efficacy of a particular type of technical progress with the diversity of price paths which it can handle while guaranteeing survival for the economy (in a particular sector). The analysis reveals that capital-saving technical progress may be preferable to resource-saving technical progress, especially when capital is more productive (see the conditions of theorem 4.4), since saving on capital is an indirect saving on the resource.

The results presented here on technical progress are in a very primitive state: in future work, we hope to model forms of technical progress which serve to radically alter the structure of production (see the discussion in section 3). The following questions may be addressed:

(1) What rates of technical progress, of the kind we have analyzed in this paper, would permit sustained growth in an economy facing steadily increasing resource prices?

(2) Given that a breakthrough of the 'structure-altering' type would solve

the energy problem but that the time of breakthrough is uncertain, what must be the allocation of funds to research and development programs aimed at developing a viable energy source such as solar energy?

(3) Given a choice between investing in 'structure-altering' and 'non-structure-altering' research and development, should one push forward slowly but surely, effecting marginal improvements of the 'non-structure-altering' kind, or support the uncertainty-laden but 'structure-altering' ventures?

(4) Given that (1)–(3) could be answered, what would be the maximum growth rates of consumption possible in such an economy after 'optimal' decisions have been arrived at regarding the nature of technical progress to be adopted (and its funding)?

These are important questions. Our analysis is at a preliminary stage, but hopefully contributes the useful first step.

Appendix

Proof of theorem 2.1. We first show that $r_t \leqq Q_t$ for all t. We consider two mutually exclusive possibilities: (i) if $E_{At} > q_t E_{Xt}$ for some t, then $p_t R_t = q_t E_{Xt} + E_{At} < 2E_{At} \leqq 2A(K_t, R_t) = 2K_t a(r_t)$. This means that $p_t/2 \leqq f(r_t)$. Under A.2.1, $f(z)$ is monotone decreasing in z, with $\lim_{z\to 0} f(z) = \infty$ and $\lim_{z\to\infty} f(z) = 0$. So $f^{-1}(\cdot)$ exists, and is monotone decreasing; hence, $r_t \leqq f^{-1}(p_t/2) \leqq Q_t$; (ii) otherwise, if $E_{At} \leqq q_t E_{Xt}$ for some t, $p_t R_t = q_t E_{Xt} + E_{At} \leqq 2q_t E_{Xt} \leqq 2q_t X(K_t, R_t) = 2q_t K_t x(r_t)$. This means that $p_t/2q_t \leqq g(r_t)$. Under A.2.1, $g(z)$ is monotone decreasing in z, with $\lim_{z\to 0} g(z) = \infty$ and $\lim_{z\to\infty} g(z) = 0$. So $g^{-1}(\cdot)$ exists, and is monotone decreasing; hence, $r_t \leqq g^{-1}(p_t/2q_t) \leqq Q_t$.

Next from (2.5) we get the inequalities: $K_{t+1} - K_t = I_t \leqq X(K_t, R_t) = K_t x(r_t) \leqq K_t x(Q_t)$ (since $x(z)$ is monotone increasing in z). This leads to $K_t \leqq K_0 \prod_{s=0}^{t-1} [1 + x(Q_s)]$.

Denote $\inf_{t\to 0} C_{At}$ by δ. Using the monotonicity of the function $a(z)$ we get the following inequalities:

$$0 < \delta \leqq C_{At} \leqq A(K_t, R_t) = K_t a(r_t) \leqq K_0 \cdot a(Q_t) \prod_{s=0}^{t-1} [1 + x(Q_s)].$$

The condition (A) is now obtained by taking reciprocals. Similarly, we derive the conditions (X). Q.E.D.

Proof of theorem 2.2. Pick $\lambda = 8$, $B = \frac{1}{8}$. We now construct the following sequence from an initial stock K_0:

(1) Let $K_t = K_0 \prod_{s=0}^{t-1} [1 + Bx(Q_s)]$ for all $t \geqq 1$; and for all $t \geqq 0$, define (2) $R_t = Q_t' K_t$; (3) $K_{Xt} = K_{At} = K_t/2$; (4) $R_{At} = R_{Xt} = R_t/2$; (5) $A_t = A(K_{At}, R_{At})$; (6) X_t

$=X(K_{Xt}, R_{Xt})$; (7) $C_{At}=E_{At}=A_t/2$ if $Q_t'=f^{-1}(\lambda p_t/2)$, $C_{At}=A_t$; $E_{At}=0$ if $Q_t'=g^{-1}(\lambda p_t/2q_t)$; (8) $I_t=X_t/4$, $E_{Xt}=X_t/2$, $C_{Xt}=X_t/4$ if $Q_t'=f^{-1}(\lambda p_t/2)$, $I_t=X_t/4$, $E_{Xt}=0$, $C_{Xt}=3X_t/4$ if $Q_t'=g^{-1}(\lambda p_t/2q_t)$. |

To verify that this sequence represents a feasible program, we must check the balance-of-payments equation and the capital accumulation equation. This is tedious but straightforward. Now,

$$C_{At}\geqq \tfrac{1}{2}A_t=\tfrac{1}{4}\cdot K_t a(r_t)=\frac{K_0}{4}\cdot a(Q_t')\prod_{s=0}^{t-1}[1+Bx(Q_s')].$$

Using the condition (A^*), we get $\inf_t C_{At}>0$.

Also,

$$C_{Xt}\geqq \tfrac{1}{4}X_t=\tfrac{1}{8}K_t x(r_t)=\frac{K_0}{8}x(Q_t')\cdot\prod_{s=0}^{t-1}[1+Bx(Q_s')].$$

Using the condition (X^*), we get $\inf_t C_{Xt}>0$. Q.E.D.

Proof of theorem 2.3. The fact that f^{-1} and g^{-1} are monotone decreasing functions of their arguments means that as p_t and $p_t/2q_t$ go to infinity, Q_t' goes to zero (recall the definition of Q_t' from (2.11)). It follows that as p_t and $p_t/2q_t$ go to infinity, so do $1/a(Q_t')$ and $1/x(Q_t')$. Theorem 2.3 will clearly follow from theorem 2.1 if we show that the conditions (A) and (X) are both violated. To this effect, it is now enough to establish the boundedness of the product $\prod_{s=0}^{t-1}[1+x(Q_s')]$. This will, in turn, follow if the series $\sum_{s=0}^{\infty}x(Q_s')$ is finite [see, e.g., Knopp (1956, p. 94)]. Thus, the final step is to prove the convergence of the infinite series $\sum_{s=0}^{\infty}x(Q_s')$ by appealing to the ratio test [see, e.g., Knopp (1956, p. 59)].

Define the sequence (θ_t) as

$$\theta_t\equiv f^{-1}(p_{t+1}/2)/f^{-1}(p_t/2). \tag{A1}$$

Note that $\theta_t<1$ for all t, since $f^{-1}(\cdot)$ is strictly decreasing and $p_{t+1}>p_t$. We shall establish that

$$\theta_t\leqq(1+g)/(1+2g)<1, \qquad \text{for all } t. \tag{A2}$$

To this effect, use the mean value theorem to get

$$f^{-1}(p_{t+1}/2)-f^{-1}(p_t/2)=\tfrac{1}{2}(p_{t+1}-p_t)f^{-1\prime}(\xi), \tag{A3}$$

where $p_t/2<\xi<p_{t+1}/2$. From (A3) and $p_0=p_t(1+g)^t$, we obtain

$$f^{-1}(p_{t+1}/2)(1-1/\theta_t)=gp_{t+1}f^{-1\prime}(\xi)/[2(1+g)]. \tag{A4}$$

Some manipulation of (A4) leads to

$$(1/\theta_t-1)\geqq[g/(1+g)][-f^{-1\prime}(\xi)\xi/f^{-1}(\xi)]\geqq g(1+g). \tag{A5}$$

The last inequality follows from the relation

$$-f^{-1\prime}(\xi)\xi/f^{-1}(\xi)=1/1-[a'(z)/a(z)/z]\geqq 1,$$

where $f(\xi)=z$.

From (A5) we directly get (A2). Similarly, define (θ_t') as

$$\theta_t'\equiv g^{-1}(p_{t+1}/2q_{t+1})/g^{-1}(p_t/2q_t), \tag{A6}$$

and show that

$$\theta_t'\leqq(1+g')/(1+2g')<1, \qquad \text{for all } t. \tag{A7}$$

Thus,

$$\begin{aligned} Q_{t+1}' &= \max[f^{-1}(p_{t+1}/2), g^{-1}(p_{t+1}/2q_{t+1})] \\ &= \max[\theta_t f^{-1}(p_t/2), \theta_t' g^{-1}(p_t/2q_t)] \\ &\leqq KQ_t', \qquad \text{where } K<1. \end{aligned} \tag{A8}$$

Since $x(\cdot)$ is strictly increasing, we define the sequence (c_t) as

$$c_t\equiv x(Q_{t+1}')/x(Q_t'), \qquad t\geqq 0, \tag{A9}$$

with $c_t<1$ for all t. Apply the mean value theorem to get

$$x(Q_t')-x(Q_{t+1}')=(Q_t'-Q_{t+1}')x'(\zeta), \qquad Q_{t+1}'<\zeta<Q_t',$$

or,

$$(1-c_t)x(Q_t')\geqq(1-K)Q_t'x'(Q_t'),$$

leading to

$$(1-c_t)\geqq(1-K)Q_t'x'(Q_t')/x(Q_t'),$$

or,

$$c_t\leqq 1-[(1-K)Q_t'x'(Q_t')/x(Q_t')].$$

Note that Q_t' goes to zero, and using the condition (2.12) we conclude that $\lim_t \sup c_t<1$. Thus, the ratio test is applicable from (A9). Q.E.D.

Proof of the assertions in section 4. Suppose $p_t = (t+2)^k$, $t \geqq 0$.

Case I. If $(1-\alpha)k/\beta > 1$, the economy is not capable of survival.

$$\prod_{s=0}^{t-1} [1 + 1/p_s^{(1-\alpha)/\beta}] = \prod_{s=0}^{t-1} \left[1 + \frac{1}{(s+2)^{(1-\alpha)k/\beta}}\right] \leqq \exp \sum_{s=0}^{t-1} \frac{1}{(s+2)^{(1-\alpha)k/\beta}}.$$

Denote $(1-\alpha)k/\beta$ by θ, and note that $f(x) = 1/x^\theta$ is a positive continuous decreasing function of x, for $x \geqq 1$.

Since $\theta > 1$, $\int_1^\infty f(x)\mathrm{d}x < \infty$, call it Ω. Now,

$$\frac{1}{(s+2)^\theta} = \frac{1}{(s+2)^\theta} [(s+2)-(s+1)] \leqq \int_{s+1}^{s+2} f(x)\mathrm{d}x, \text{ for } s \geqq 0.$$

So,

$$\sum_{s=0}^{t-1} \frac{1}{(s+2)^\theta} \leqq \int_1^2 f(x)\mathrm{d}x + \cdots + \int_t^{t+1} f(x)\mathrm{d}x \leqq \int_1^\infty f(x)\mathrm{d}x = \Omega.$$

Thus,

$$\prod_{s=0}^{t-1} [1 + 1/p_s^{(1-\alpha)/\beta}] \leqq \mathrm{e}^\Omega, \qquad \text{for all } t.$$

Hence,

$$\frac{p_t^{(1-\beta)/\beta}}{\prod_{s=0}^{t-1} [1 + 1/p_s^{(1-\alpha)/\beta}]} \geqq \frac{p_t^{(1-\beta)/\beta}}{\mathrm{e}^\Omega} \to \infty \quad \text{as} \quad t \to \infty,$$

and

$$\frac{p_t^{(1-\alpha)/\beta}}{\prod_{s=0}^{t-1} [1 + 1/p_s^{(1-\alpha)/\beta}]} \geqq \frac{p_t^{(1-\alpha)/\beta}}{\mathrm{e}^\Omega} \to \infty \quad \text{as} \quad t \to \infty.$$

Hence, by theorem 4.1, the economy is not capable of survival.

Case II. If $(1-\alpha)k/\beta < 1$, the economy is capable of survival. For $y \leqq 2$, $\mathrm{e}^y - \mathrm{e}^0 \leqq \mathrm{e}^2 y$ (using mean value theorem); or $\mathrm{e}^y \leqq 1 + \mathrm{e}^2 y$. Now, given $L \in (0,1]$, choose $d = L/\mathrm{e}^2$. For $s \geqq 0$,

$$\exp \frac{d}{(s+2)^\theta} \leqq \left[1 + \mathrm{e}^2 \frac{d}{(s+2)^\theta}\right] = \left[1 + \frac{L}{(s+2)^\theta}\right]$$

$$\left(\text{since } \frac{d}{(s+2)^\theta} \leqq 2 \quad \text{for} \quad s \geqq 0\right).$$

Hence,

$$\prod_{s=0}^{t-1}\left[1+\frac{L}{(s+2)^{\theta}}\right] \geqq \exp \sum_{s=0}^{t-1} \frac{d}{(s+2)^{\theta}}.$$

Now consider $g(x)=d/x^{\theta}$. This is a positive, continuous, decreasing function of x for $x \geqq 1$.

$$\frac{d}{(s+2)^{\theta}}=\frac{d}{(s+2)^{\theta}}[(s+3)-(s+2)] \geqq \int_{s+2}^{s+3} g(x)\mathrm{d}x,$$

$$\sum_{s=0}^{t-1} \frac{d}{(s+2)^{\theta}} \geqq \int_{2}^{3} g(x)\mathrm{d}x+\cdots+\int_{t+1}^{t+2} g(x)\mathrm{d}x=\int_{2}^{t+2} g(x)\mathrm{d}x=\int_{2}^{t+2} d/x^{\theta}\mathrm{d}x$$

$$=\left.\frac{dx^{1-\theta}}{1-\theta}\right]_{2}^{t+2}=\frac{d(t+2)^{(1-\theta)}}{1-\theta}-\frac{d2^{(1-\theta)}}{1-\theta}=m(t+2)^{(1-\theta)}-C,$$

where $m=d/(1-\theta)$, $C=d2^{(1-\theta)}/\theta$. Hence,

$$\prod_{s=0}^{t-1}\left[1+\frac{L}{(s+2)^{\theta}}\right] \geqq \exp \sum_{s=0}^{t-1} \frac{d}{(s+2)^{\theta}} \geqq \exp\left[m(t+2)^{(1-\theta)}-C\right].$$

So,

$$\frac{p_t^{(1-\alpha)/\beta}}{\prod_{s=0}^{t-1}\left[1+\frac{L}{p_s^{(1-\alpha)/\beta}}\right]} \leqq \frac{(t+2)^{\theta}}{\exp\left[m(t+2)^{(1-\theta)}-C\right]} \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty,$$

and

$$\frac{p_t^{(1-\beta)/\beta}}{\prod_{s=0}^{t-1}\left[1+\frac{L}{p_s^{(1-\beta)/\beta}}\right]} \leqq \frac{(t+2)^{\theta \cdot \frac{1-\beta}{1-\alpha}}}{\exp\left[m(t+2)^{1-\theta}-C\right]} \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty.$$

Hence by theorem 4.2, there exists a feasible program with $\inf_{t \geqq 0} C_{At}>0$ and $\inf_{t \geqq 0} C_{Xt}>0$. Q.E.D.

To construct the feasible program referred to in section 4 choose $0<\lambda<1$, such that $1/\lambda^{\beta}=4$. Denote $\lambda^{1-\alpha}/4$ by L (recall that $\alpha=\beta=\frac{1}{2}$). Construct the feasible program from K_0 as follows:

$$K_t=K_0 \prod_{s=0}^{t-1}\left[1+\frac{L}{p_s^{(1-\alpha)/\beta}}\right] \quad \text{for} \quad t \geqq 1;$$

$$R_t = \lambda K_t / p_t^{(1/\beta)} \quad \text{for} \quad t \geqq 0; \qquad R_{Xt} = R_{At} = R_t/2;$$

$$K_{Xt} = K_{At} = K_t/2; \qquad A_t = K_{At}^{\beta} R_{At}^{1-\beta}; \qquad X_t = K_{Xt}^{\alpha} R_{Xt}^{1-\alpha};$$

$$C_{At} = E_{At} = A_t/2; \qquad C_{Xt} = I_t = X_t/2.$$

In order to verify that this is a feasible program, we have only to check the balance of payments equation, and the capital accumulation equation. This is left as an exercise.

Proof of theorem 4.3. For simplicity of computations leading to the bound (A11) derived below, we proceed by setting $\theta = 0$ in (4.9). It is, of course, intuitive that if $\theta > 0$, i.e. if capital stock does depreciate at a fixed rate, the stock of capital at any period along a feasible program cannot be higher than that attained under the assumption of no depreciation. Formal derivation of (A11) when $\theta > 0$ is left as an exercise. With $\theta = 0$, we get $K_{t+1} \geqq K_t$ (from (4.9)) and $K_{t+1} - K_t \leqq K_t^{\alpha} R_t^{1-\alpha}$ (from (4.2)), and directly verify that

$$K_{t+1}^{1-\alpha} - K_t^{1-\alpha} \leqq \frac{K_{t+1}}{K_t^{\alpha}} - \frac{K_t}{K_t^{\alpha}} \leqq R_t^{1-\alpha},$$

or,

$$K_{t+1}^{1-\alpha} - K_0^{1-\alpha} \leqq \sum_{s=0}^{t} R_s^{1-\alpha} \leqq \left[\sum_{s=0}^{t} R_s \right]^{1-\alpha} (t+1)^{\alpha}$$

(by Holder's inequality).

Hence, there is some constant $J > 0$ such that

$$K_{t+1}^{1-\alpha} \leqq J \left[\sum_{s=0}^{t} R_s \right]^{1-\alpha} (t+1)^{\alpha}. \tag{A10}$$

Denote $\sum_{s=0}^{t-1} R_s$ by D_t. Then

$$K_t \leqq a D_t t^{\alpha/1-\alpha}, \qquad \text{where } a \equiv J^{1/1-\alpha}. \tag{A11}$$

From (4.9) we have the basic stock inequality

$$S_t \leqq S_{t-1} + M_t - R_t. \tag{A12}$$

For each t, there are two possibilities to be examined separately: Case I, $M_t < R_t/2$; and Case II, $M_t \geqq R_t/2$. If for a particular t, Case I occurs, then $S_t \leqq S_{t-1}$. Otherwise, Case II occurs, and we note that

$M_t \leqq A_t/p_t \leqq K_t^{\beta} R_t^{1-\beta}/p_t \leqq K_t^{\beta} 2^{1-\beta} M_t^{1-\beta}/p_t$. Using (A11) and $p_t = p_0(1+g)^t$, we get

$$M_t \leqq \hat{a} D_t/(1+b')^t, \qquad \text{for some constants } \hat{a}>0,\ g>b'>0. \qquad \text{(A13)}$$

From (A12) and (A13), in Case II, $S_t \leqq S_{t-1} + \hat{a} D_t/(1+b')^t$. Thus, we conclude that we always get, for each t,

$$S_t \leqq S_{t-1} + \hat{a} D_t/(1+b')^t$$

or,

$$S_t - S_0 \leqq \sum_{s=0}^{t} \hat{a} D_s/(1+b')^s \leqq \hat{b} D_t, \text{ for some constant } 0<\hat{b}<\infty. \qquad \text{(A14)}$$

Going back to Case I, and using (A11), we have: $R_t/2 \leqq S_{t-1} \leqq \hat{b} D_{t-1} \leqq \hat{b} D_t$, implying $R_t \leqq 2\hat{b} D_t$. Now, $M_t \leqq A_t/p_t \leqq (m D_t \cdot t^{\alpha\beta/1-\alpha})/p_t$, where m is a positive constant. Hence, in Case I, there is some finite $\hat{m}>0$ such that

$$M_t \leqq \hat{m} D_t/(1+b'')^t, \qquad \text{where } 0<b''<g. \qquad \text{(A15)}$$

Let $\bar{m} = \max(\hat{a}, \hat{m})$, $b = \min(b', b'')$. From (A13) and (A15),

$$M_t \leqq \bar{m} D_t/(1+b)^t, \qquad \text{for all } t. \qquad \text{(A16)}$$

Our next step is to prove the following:

$$\text{The sequence } D_t \equiv \sum_{s=0}^{t-1} R_s \text{ is bounded above.} \qquad \text{(A17)}$$

Suppose to the contrary that D_t goes to infinity with t. Define $\varepsilon = b/4(1+b)$, and T such that $\bar{m}/(1+b)^T < \varepsilon$. Then for $t \geqq T$, $M_t \leqq [\bar{m}/(1+b)^T] D_t/(1+b)^{t-T} \leqq \varepsilon D_t/(1+b)^{t-T}$. Choose $T^* > T$, large enough so that for all $\tau \geqq T^*$, $D_T/D_\tau \leqq D_{T^*} < \frac{1}{4}$ (since D_t is a monotonically nondecreasing sequence which is assumed to be unbounded, this choice is possible). Hence, $\sum_{t=T}^{\tau-1} R_t \equiv D_\tau - D_T \geqq \frac{3}{4} D_\tau$. Also, from (A16) verify that $\sum_{t=T}^{\tau-1} M_t \leqq \varepsilon D_{\tau-1}(1+b)/b = \frac{1}{4} D_{\tau-1}$. Hence, for $\tau > T^*$,

$$\sum_{t=T}^{\tau-1} [S_t - S_{t-1}] \leqq \sum_{t=T}^{\tau-1} M_t - \sum_{t=T}^{\tau-1} R_t,$$

or,

$$S_{\tau-1} - S_{T-1} \leqq -D_\tau/2.$$

But, as $D_\tau \to \infty$ by hypothesis, we are led to $S_{\tau-1} < 0$ for large τ, a contradiction to the non-negativity of stocks. This establishes (A17). It follows that the sequence D_t necessarily converges, and let the limit be denoted by $\bar{D} \equiv \sum_{s=0}^{\infty} R_s$.

We are now prepared to show that the economy is not capable of survival. Consider, first, the case $\delta > 0$. Suppose that for some feasible program $\inf_{t \geq 0} C_{Xt} = \gamma > 0$. This means that $K_t^\alpha R_t^{1-\alpha} \geqq C_{Xt} \geqq \gamma$ for all t. From (A11) and (A16), we derive a positive constant J' such that

$$R_t \geqq (\gamma / K_t^\alpha)^{1/1-\alpha} \geqq J'/t^{\alpha^2/1-\alpha}. \tag{A18}$$

From (A16), there is some $\bar{T}$ such that $R_t > M_t$ for all $t \geqq \bar{T}$. Hence, $S_t \leqq (1-\delta)S_{t-1}$ for $t \geqq \bar{T}$, implying $S_t \leqq (1-\delta)^{t-T} S_{\bar{T}}$. But, this means that $\lim_{t\to\infty} S_t = 0$. From (4.9)

$$R_t = (1-\delta)S_{t-1} + M_t - S_t \leqq (1-\delta)S_{t-1} + M_t,$$

and using (A16), $\lim_{t\to\infty} R_t = 0$, a contradiction to (A18). This proves that for any feasible program $\inf_{t \geqq 0} C_{Xt} = 0$, and an analogous argument can be used to show that there is no feasible program with $\inf_{t \geqq 0} C_{At} > 0$.

Finally, consider the case $\theta > 0$. Note that in such a case, for any feasible program K_t must be bounded above. Suppose to the contrary that there is a subsequence of time periods $\langle t_s \rangle$ such that K_{t_s} goes to infinity with t_s. We show that $K_t < 1$ for only a finite number of time periods. If not, there is a subsequence $\langle t_r \rangle$ with $K_{t_r} < 1$. Choose T large enough so that $R_t^{1-\alpha} < \theta$ for all $t \geqq T$ (since $\sum_{t=0}^{\infty} R_t = \bar{D}$ this can be done). Let $\bar{t}_r$ be the first term of the subsequence such that $\bar{t}_r \geqq T$. Then,

$$K_{\bar{t}_r+1} \leqq (1-\theta)K_{\bar{t}_r} + K_{\bar{t}_r}^\alpha R_{\bar{t}_r}^{1-\alpha} < (1-\theta) + R_{\bar{t}_r}^{1-\alpha} < 1.$$

The same argument leads to $K_t < 1$ for all $t \geqq \bar{t}_r$, a contradiction to the supposition that K_{t_s} goes to infinity along the subsequence $\langle t_s \rangle$. Thus, there is some $\hat{T} > T$ such that $K_t \geqq 1$ for all $t \geqq \hat{T}$. Then,

$$\begin{aligned} K_{t+1} &\leqq (1-\theta)K_t + K_t^\alpha R_t^{1-\alpha} \leqq (1-\theta)K_t + K_t R_t^{1-\alpha} \\ &= K_t[(1-\theta) + R_t^{1-\alpha}] < K_t, \end{aligned}$$

for all $t \geqq \hat{T}$. This, again, contradicts the hypothesis that K_{t_s} tends to infinity. Thus, the sequence K_t is bounded above, i.e. for some $\bar{K} > 0$, $K_t \leqq \bar{K}$ for all t.

Now, $0 \leqq C_{Xt} \leqq K_t^\alpha R_t^{1-\alpha} \leqq \bar{K}^\alpha R_t^{1-\alpha}$ and, since R_t goes to zero as $t \to \infty$, we have $\lim_{t\to\infty} C_{Xt} = 0$. Similarly, we can prove that $\lim_{t\to\infty} C_{At} = 0$. Thus, the economy is not capable of survival. Q.E.D.

References

Bhagwati, J. and R.S. Eckaus, eds., 1970, Foreign Aid (Penguin, Harmondsworth).
Bhagwati, J., 1964, The pure theory of international trade: A survey, Economic Journal 74, Reprinted in Surveys of Economic Theory, vol. 2, Macmillan, 1965, pp. 156–239.
Cass, D. and T. Mitra, 1981, On maintaining minimal consumption levels in the presence of exhaustible resources, Journal of Economic Theory.
Chenery, H. and M. Bruno, 1962, Development alternatives in an open economy: The case of Israel, Economic Journal LXXII, no. 285, 79–103.
Dasgupta, P. and G. Heal, 1974, The optimal depletion of exhaustible resources, Review of Economic Studies, Symposium on Exhaustible Resources, 3–28.
Dasgupta, P., G. Heal and M. Majumdar, 1977, Resource depletion, research and development, in: M.D. Intriligator, ed., Frontiers of Quantitative Economics, Vol. IIIb, 483–505.
Davison, R., 1978, Optimal depletion of an exhaustible resource with research and development towards an alternative technology, Review of Economic Studies XLV(2), no. 140, 355–367.
Hicks, J.R., 1953, An inaugural lecture, Oxford Economic Papers 5, no. 2, 117–135.
Hotelling, H., 1931, The economics of exhaustible resources, Journal of Political Economy 39, 137–175.
Kemp, M.C. and H. Suzuki, 1975, International trade with a wasting but possibly replenishable resource, International Economic Review, 712–732.
Knopp, K., 1956, Infinite sequences and series (Dover, New York).
Lewis, W.A., 1954, Economic development with unlimited supplies of labour, The Manchester School, reprinted in A.N. Agarwala and S.P. Singh, eds., The Economics of Underdevelopment, 400–439.
Malinvaud, E., 1969, Capital accumulation and efficient allocation of resources, Reprinted in: K.J. Arrow and T. Scitovksy, eds., Readings in Welfare Economics, 645–681.
Ramsey, F.P., 1928, A mathematical theory of savings, Economic Journal 38, 543–559.
Prebisch, R., 1959, Commercial policy in the underdeveloped countries, American Economic Review, supplement to vol. 49, Papers and Proceedings, 251–273.
Robinson, J., 1958, The classification of inventions, Review of Economic Studies V, no. 2, 139–142.
Singer, H., 1950, The distribution of gains between investing and borrowing countries, American Economic Review, supplement to vol. 40, Papers and Proceedings, 473–485.
Solow, R.M., 1974, Intergenerational equity and exhaustible resources, Review of Economic Studies, Symposium on Exhaustible Resources, 29–45.
Stiglitz, J.E., 1974, Growth with exhaustible natural resources: Efficient and optimal growth paths, Review of Economic Studies, Symposium on Exhaustible Resources, 123–137.
Von Neumann, J., 1945–6, A model of general economic equilibrium, Review of Economic Studies XIII(1), no. 33, 1–9.
Vousden, N., 1974, International trade and exhaustible resources: A theoretical model, International Economic Review, 149–167.