Aspirations, Segregation and Occupational Choice

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Abstract

This paper examines steady states of an overlapping generations economy with a given distribution of household locations over a one-dimensional interval. Parents decide whether or not to educate their children. Such decisions are affected by location: parental aspirations depend on the earnings of their neighbors. At the same time, economy-wide wages endogenously adjust to bring factor supplies into line with demand. The model therefore combines local social interaction with global market interaction. The paper studies steady-state configurations of skill acquisition, both with and without segregation, and studies the macroeconomic and welfare effects of segregation on aggregate economic outcomes.

1. Introduction

This paper introduces parental aspirations into a theory of occupational choice. Parental aspirations are generated by the economic achievements of neighbors. At the same time, wages adjust endogenously to equate occupational supply and demand. The paper therefore combines localized social interaction with global market interaction.

A recent literature on occupational choice (going back to Loury 1981; Ray 1990; Banerjee and Newman 1993; Galor and Zeira 1993) emphasizes capital market imperfections and the importance of parental wealth in determining educational investment. While we work within this framework, our main concern is to understand how geography *per se* — the income and skill characteristics of local neighborhoods — interact with wage incentives to determine an equilibrium supply of skilled labor. While we model such local effects in a simple way by having them shape parental aspirations, the same model could serve as a proxy for the direct influence of neighbors and peers in shaping motivations, or access to local public goods or learning opportunities. Similarly, while we adopt a simplistic, spatial notion of "neighborhood" in our model, the concept can easily be extended to span both physical and social distance. For instance, social networks based on ethnicity, race or caste can play a role analogous to those of one's physical neighbors.

While various patterns of spatial segregation are indeed possible in our model, it turns out that the *combination* of endogenous factor prices and local interaction imposes a lot of structure on equilibrium outcomes. Under some restrictions, we are able to fully describe equilibrium patterns of geographical segregation, in which human capital investment varies across neighborhoods or social groups in a persistent fashion.

It is undeniable that similar patterns of segregation are commonly observed in developed and developing countries alike: inner-city decay, slums coexisting side-by-side with affluent gated communities, regional inequality, and unequal levels of educational achievement across race, ethnic or caste lines. Now, some of this is surely explained by capital market imperfections, combined with the tendency for unskilled and skilled agents in an economy to cluster geographically (e.g., owing to variations in housing prices, local congestion or simply the preference of skilled agents to locate near other skilled people). Under this view, the observed geographical patterns are a consequence of economic inequality combined with the spatial mobility of agents.

In contrast, we want to examine whether the reverse is possible, i.e., whether geography can be a primary determinant of economic inequality, in the absence of capital market imperfections and agent mobility. Under this second view, different patterns of economic segregation will be associated with different macroeconomic outcomes, because variations in these patterns directly influence the decision to invest. Thus it is natural to investigate, in such a model, the associations between persistent geographic inequality (or segregation, for short) and macroeconomic and welfare outcomes. This is why we deliberately abstract from capital-market imperfections or the spatial mobility of economic agents. This distinguishes

our analysis from a number of previous models of segregation that result from mobility, e.g., Schelling (1978), Bénabou (1993) or more recently Pancs and Vriend (2007).[1]

In the end, the extent to which agents are able to move (the cost of mobility) is an empirical question on which we do not comment. We simply study the implications of immobility, under the presumption that this assumption is just as plausible as perfect mobility (depending on the situation), and in many cases, more plausible. For instance, less developed countries exhibit substantially less spatial mobility than their developed counterparts. Bardhan (2002) argues that the assumption of zero mobility costs underlying the Tiebout model of local public goods is empirically implausible for most developing countries.[2]

Our model, then, has the following features. There is a large number of families in which parents make a binary skill investment in their children. Wages for skilled and unskilled labor settle to equate supply and demand for these factors. This is a standard model of occupational choice, and it easily can (and often does) incorporate capital market imperfections. In order to focus on geographical effects *per se*, for the most part we consider a version where parental utility for consumption is linear, so rich and poor parents located in a given neighborhood have the same incentives.[3]

Into this standard formulation we introduce parental aspirations. We describe such aspirations as targets or benchmarks for the material earnings of their children. The higher the benchmark, the greater the parental motivation to invest in the child (controlling for wage differentials). Aspirations, in turn, are based on the current or past achievements of one's neighbors, located within some exogenously given spatial or social window.

In a steady state, the investment decisions of all agents *and* their aspirations will be endogenously determined, as well as market wages. Many features of our model will be driven by the interaction of the economic (economy-wide) externality with the social (local) externality. This is a distinct feature that differentiates our model from most of the existing literature on segregation.[4]

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[1] In any case, if mobility costs were really zero, geographic segregation or housing prices (or local congestion or school quality) would not be associated with welfare inequality among agents with the same wealth. Differences in housing costs or congestion would fully reflect differences in school quality in equilibrium. Any utility differences across neighborhoods would generate mobility and get "arbitraged away". In Benabou's model, this is precisely the nature of the equilibrium. He assumes the stock of housing in each neighborhood to be given, so different patterns of segregation have implications for rents earned by passive landlords, and inequality between landlords and tenants.

[2] On the other hand, spatial mobility may be replaced by other forms of clustering which can have significant implications for human capital accumulation and wealth inequality across families. An excellent example of this is marital sorting (see Fernández and Rogerson 2001 and Fernández 2003).

[3] Our dynamic formulation with overlapping generations is motivated by our interest in models of human capital in which parents make investment decisions on behalf of their children. An analogous static model (where a given generation of agents make their own investment decisions) would deliver similar macroeconomic outcomes but substantially distinct welfare properties, in the absence of the intra-family parent-child externality.

[4] After a first draft of this paper was completed, we became aware of independent work by Ghiglino and Goyal (2008), which studies local effects in the consumption of status goods within a static model. In particular, they emphasize the effect of exogenous social networks — specifically, the positioning of high-wealth, high-status consumers in that network — on equilibrium prices, allocations and welfare. Therefore, like us, Ghiglino and Goyal also combine local interactions with endogenous market-clearing prices. But there are many differences

As we've already mentioned, a broader variety of neighborhood effects could also be incorporated, such as peer influences and local role-models, as well as local effects that affect the (personal) cost of educational investments, such as access to quality schools or opportunities to learn from one's neighbors. For the most part, the essential assumption is one of complementarity at the local level: higher earnings of neighbors increase the incentive to invest.

Similarly, our formulation of social preferences is related to but not the same as models of identity (Akerlof and Kranton 2000; 2005) or conformity (Bisin et al. 2006), where well-being depends negatively on the distance between one's own *actions* and those of one's neighbors. Not only are preferences in those models defined directly over actions, but the main concern is for conformity. In contrast, agents in the aspirations-based version of our model are happiest if they can differentiate themselves favorably from their peers. Nevertheless, given the complementarity property it induces, the results of our model concerning existence and characterization would be unchanged if we were to use a conformity-based formulation instead.

It will turn out, however, that the precise source of neighborhood effects does matter when we evaluate the welfare consequences of segregation. Welfare properties depend on how neighborhoods affect the *level* of utility, while the structure of equilibria depend only on how they affect investment incentives (i.e., *marginal* utility).

Section 2 introduces a model of occupational choice, one that allows both for the existence of capital-market imperfections and for socially-determined parental aspirations. Definitions of segregated and unsegregated steady state equilibria (or equilibria for short) are provided. Section 3 shows that segregated and unsegregated equilibria both exist in general. Section 4 specializes to the case of linearity of parental utility in own consumption, where the capital market imperfection is effectively absent. The three subsections that follow (4.1–4.3) derive specific properties of segregated and unsegregated equilibria in this setting. Subsection 4.4 then compares macroeconomic properties of the two classes of equilibria, while Section 5 compares their welfare effects. Finally, Section 6 summarizes the main results and discusses issues that need to be addressed in future work.

2. The Model

2.1. **Locations and Skills.** A unit mass of households is exogenously located on some compact interval $I = [\underline{\iota}, \overline{\iota}]$ of the real line, and their locational distribution is described by a continuous density function f on I. We presume that f is strictly positive everywhere in the interior of I, that it is nowhere flat, and that it has a finite number of turning points. These assumptions imply, in particular, that there are finitely many locations $i_1, \dots, i_{K+1}$, with $\underline{\iota} = i_1 < \cdots < i_{K+1} = \overline{\iota}$ such that f is strictly monotone on each interval $[i_k, i_{k+1}]$ and alternately increasing and decreasing across consecutive intervals I_k and I_{k+1}. We shall refer to these "pieces" of f as (increasing or decreasing) *stretches*.

in questions, emphasis and modeling. For instance, segregation patterns are (to some extent) endogenous in this paper and so is wealth inequality. Moreover, Ghiglino and Goyal deal with a different form of social interdependence; namely, competitive consumption à la Veblen-Frank. In future work, it would be interesting to explore its effects in our setting (see footnote 15 below).
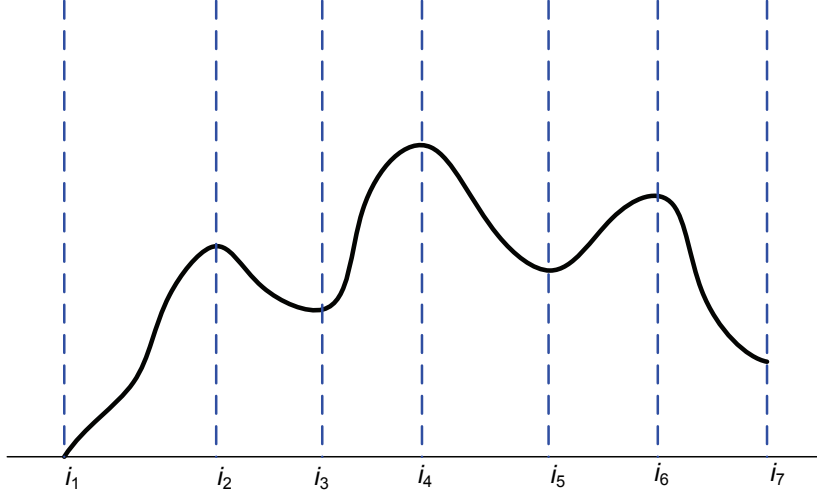
FIGURE 1. A DISTRIBUTION OF HOUSEHOLDS

For notational convenience, we define $f(i) = 0$ elsewhere on $\mathbb{R}$ (the extension need not be continuous at the edges of I).

Figure 1 illustrates a typical distribution f, as well as its stretches.

At any location i there are households (with density $f(i)$) with each household consisting of a single active agent (the parent) at any date. The agent is either *skilled* or *unskilled*. The indicator function $\mathbf{1}(h)$ stands for household h being skilled, and $\beta(i)$ is just the average of these indicators at location i. The overall skill ratio in society is therefore given by

$$\lambda = \int_I \beta(i)f(i)dx.$$

2.2. **Wages.** The going skill ratio λ determines skilled and unskilled wages, denoted by $\bar{w}(\lambda)$ and $\underline{w}(\lambda)$, in the society at large. This represents the economy-wide market interaction among households, arising from imperfect substitutability between skilled and unskilled labor in the technology, and the assumption that the labor market is integrated throughout the economy. Specifically, we presume that these wages are the marginal products of a production technology, described by a continuously differentiable, constant-returns-to-scale, strictly quasiconcave, Inada production function T defined on skilled and unskilled labor:

$$\bar{w}(\lambda) = T_1(\lambda, 1 - \lambda) \text{ and } \underline{w}(\lambda) = T_2(\lambda, 1 - \lambda),$$

where subscript j, $j = 1, 2$, denotes the derivative with respect to the jth input.

It follows that $\bar{w}(\lambda)$ and $-\underline{w}(\lambda)$ are continuous and strictly decreasing, that the end-point conditions $\lim_{\lambda \downarrow 0} \bar{w}(\lambda) = \infty$ and $\lim_{\lambda \downarrow 0} \underline{w}(\lambda) = 0$ are satisfied, and that there is some value $\bar{\lambda} \in (0, 1)$ with $\bar{w}(\bar{\lambda}) = \underline{w}(\bar{\lambda})$.[5]

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[5]Our assumptions imply that the wage differential to the right of $\bar{\lambda}$ is negative, but behavior in this region is unimportant as long as the wage differential does not turn strictly positive again. For instance, if skilled

2.3. **Cognitive Windows, Aspirations and Skill Choices.** In line with dynamic models of occupational choice, we view the single agent in each household as replaced in every period by a member of a new generation, with parent and child linked by intergenerational altruism. Specifically, each parent makes the skill choice for her descendant. We normalize so that to create an unskilled descendant costs nothing, while a skilled child costs x, an exogenous price that is incurred by the parent. Thus a parent in household h with going wage w obtains a direct utility of $u\,(w - \mathbf{1}'(h)x)$, where the indicator function $\mathbf{1}'$ refers now to the distribution of skills in the "next period".

In addition, she receives an indirect utility v by educating her child. The central two-part premise of this paper is that v is fundamentally affected by parental aspirations, while such aspirations in turn are determined by the distribution of wages in the parent's local cognitive window. We capture the first of these points by introducing a scalar a called parental aspirations, and assume that indirect utility depends on descendant wages w' as well as aspirations: $v(w', a)$. Thus overall parental utility is assumed to be given by

$$u\,(w - \mathbf{1}'(h)x) + v(w', a),$$

where we shall presently place restrictions on u and v.

The second part of our central premise relates aspirations to the neighborhood wage distribution in the parent's cognitive window. Indeed, parental aspirations could be determined in different ways. They might be "private" — some function of *that* household's wage history, or they might be "social" — some average of wages in the neighborhood or in the economy as a whole. In this paper we study social aspirations in which such aspirations are drawn from a local window of neighbors (but we wish to emphasize that other formulations are indeed possible).

We study, therefore, the case in which $a = a(i)$ for every household at location i, which in turn is just the unweighted average of wages earned by all neighbors in an ϵ-neighborhood centered at i. For any interval $N = [j, j'] \subseteq \mathbb{R}$, define $F(N) \equiv \int_j^{j'} f(i)di$; then

(1)
$$a(i) = \frac{1}{F(N)} \int_N w(j) f(j) dj,$$

where N is the interval $[i - \epsilon, i + \epsilon]$,[6] and $w(j)$ is average wage at location j:

$$w(j) = \beta(j)\bar{w} + [1 - \beta(j)]\underline{w}.$$

Now, ϵ is potentially compatible with a "global" or "local" view of aspirations (e.g., if large enough, all have the same aspirations), but we will concentrate on the local case. In particular, we will later study the limit case in which ϵ goes to zero.

Now that we have a concrete description of aspirations, we impose further restrictions on preferences. We assume that v is continuous, increasing and unbounded in its first argument.

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individuals can do unskilled jobs, it might make sense to assume that the wage differential is exactly 0 to the right of $\bar{\lambda}$.

[6]Recall that we've extended f to all of $\mathbb{R}$ so there is no ambiguity in this definition at the edges of I; households near to or at the edge see few or no individuals to one side.

The important restriction is complementarity: we assume that for any pair of wages $\underline{w} < \bar{w}$, the difference

$$v(\bar{w}, a) - v(\underline{w}, a)$$

is increasing in a. In short, higher neighborhood wages always increase the marginal incentive to invest in one's own child. A particular version of the v function is where it is a (increasing, strictly concave) function solely of the difference between the earnings of one's child and the parent's aspiration.

This is a strong assumption, and it is easy to imagine situations in which it is not satisfied. For instance, Ray (1998, Chapter 3; 2006) has argued that extremely high aspirations could be detrimental to investment, simply because it may be very difficult to catch up. That argument cannot be fully incorporated into the current model because we work only with two skill levels, so that a single educational investment does, indeed, permit full catchup. Nevertheless, the complementarity assumption is a strong one because it rules out "frustration", and in future work involving several occupational choices we plan to drop this restriction.

On the direct utility u, we impose standard assumptions: that it is increasing and concave. For simplicity of exposition, we suppose further that u is devoted over both positive and negative consumptions.

2.4. **Steady State Equilibrium.** A *steady state equilibrium* (or just *equilibrium*, for short) is a distribution **1** of skills on households, an aggregate skill ratio λ, and wages for skilled and unskilled labor ($\bar{w}$ and $\underline{w}$) such that

(i) wages are consistent with the aggregate skill ratio: $\bar{w} = \bar{w}(\lambda)$ and $\underline{w} = \underline{w}(\lambda)$;

(ii) the aggregate skill ratio is consistent with the distribution of skills: $\lambda = \int_I \beta(i) f(i) dx$, where $\beta(i)$ is the average of **1** for all households located at i;

(iii) skill choices are time-stationary and consistent with aspirations: for each h located at i, $\mathbf{1}'(h) = \mathbf{1}(h)$ solves the problem

$$\max_{\mathbf{1}'(h)} u\left(w^h - \mathbf{1}'(h)x\right) + v\left(\mathbf{1}'(h)\bar{w} + [1 - \mathbf{1}'(h)]\underline{w}, a^h\right),$$

where $w^h = \mathbf{1}(h)\bar{w} + [1 - \mathbf{1}(h)]\underline{w}$, and $a^h = a(i)$ solves equation (1).

Thus in steady state equilibrium, no household changes its choices over generations, and the distributions of skills, incomes and aspirations remains unchanged.

An equilibrium is (purely) *unsegregated* if in addition to the equilibrium conditions, aspirations do not change with location: $a(i)$ is a constant for all $i \in I$. An example of such an equilibrium is one in which skilled individuals are distributed uniformly across all locations — in every subinterval of I, no matter how small, the proportion of skilled individuals is the same.

An equilibrium is (purely) *segregated* if the distribution of skills **1** takes on values of 0 and 1 over successive intervals, with each interval at least of size ϵ. This last requirement is related to the qualification "purely". It insists that the extent of the segregation be at least as
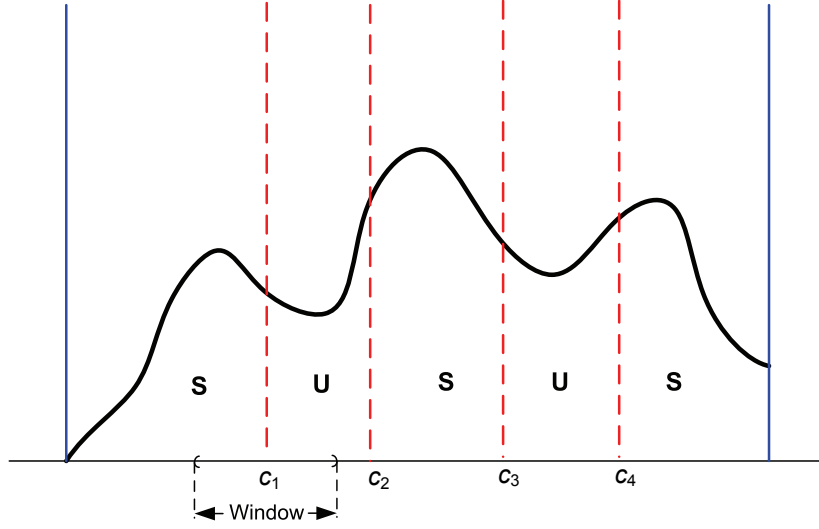
FIGURE 2. A PURELY SEGREGATED EQUILIBRIUM

large so that no individual positioned on the edge of an interval can see both skill types on any *one* side. (Of course, it is impossible to avoid seeing different skills on the two different sides.) Thus in a purely segregated equilibrium the indicator function **1** carves up I into a succession of skilled and unskilled intervals, but we are asking for more: we demand that these segments be sizeable relative to individual cognitive windows.

Figure 2 illustrates a purely segregated equilibrium where the "cuts" $c_1, \ldots c_4$ divide up the society into successive segments of skilled and unskilled households.

Thus one implication buried inside the concept of pure segregation is that we are looking at cognitive windows that are suitably small relative to the economy. But the requirement is a bit stronger than that: even for small ϵ, there could exist equilibria with skilled-unskilled intervals that are even tinier. Such equilibria have characteristics similar to those of an unsegregated equilibrium over some neighborhoods; hence we do not call them 'purely' segregated. We neglect such equilibria in the rest of this paper, as they involve a number of delicate technical issues. We defer their analysis to future research.

3. THE EXISTENCE OF UNSEGREGATED AND SEGREGATED EQUILIBRIA

In general, both segregated and unsegregated equilibria exist. Let's review the main steps that guarantee existence. First, for any skill ratio λ and any aspiration a, define

$$\Psi(\lambda, a) \equiv v\left(\bar{w}(\lambda), a\right) - v\left(\underline{w}(\lambda), a\right).$$

By the complementarity assumption, Ψ is increasing in a and by our assumptions on the wage functions, it is declining in λ.

In an unsegregated equilibrium with aggregate skill ratio λ, it must be the case that

(2) $$a = \lambda \bar{w}(\lambda) + (1 - \lambda)\underline{w}(\lambda),$$

while the following condition ensures that skilled parents will choose to invest in their children's education, while unskilled parents do not, so that we do have a steady state:

$$(3) \qquad u\left(\bar{w}(\lambda)\right) - u\left(\bar{w}(\lambda) - x\right) \leq \Psi(\lambda, a) \leq u\left(\underline{w}(\lambda)\right) - u\left(\underline{w}(\lambda) - x\right).$$

Indeed, conditions (2) and (3) are both necessary and sufficient for λ to be the outcome of a segregated equilibrium.

PROPOSITION 1. *An unsegregated equilibrium exists.*

Proof. To show that (2) and (3) must hold for some λ, define $a(\lambda)$ by the right hand side of (2), and then define $\Phi(\lambda) \equiv \Psi\left(\lambda, a(\lambda)\right)$. It is easy to see that Φ is continuous for all $\lambda \in (0, 1)$.

By Euler's theorem, $a(\lambda) = T(\lambda, 1 - \lambda)$, so it is bounded in λ. Moreover, v is unbounded in w, so it follows from the end-point conditions on $\bar{w}(\lambda)$ that $\Phi(\lambda) \to \infty$ as $\lambda \to 0$. On the other hand, it is easy to see that $\Phi(\lambda) \to 0$ as $\lambda \to \bar{\lambda}$. At the same time, $u\left(\bar{w}(\lambda)\right) - u\left(\bar{w}(\lambda) - x\right)$ is bounded, strictly positive and continuous on $(0, \bar{\lambda}]$.

Putting all these observations together, we must conclude that there exists $\lambda \in (0, \bar{\lambda})$ such that

$$(4) \qquad u\left(\bar{w}(\lambda)\right) - u\left(\bar{w}(\lambda) - x\right) = \Phi(\lambda).$$

Because u is concave and $\lambda \leq \bar{\lambda}$, we know that

$$(5) \qquad u\left(\bar{w}(\lambda)\right) - u\left(\bar{w}(\lambda) - x\right) \leq u\left(\underline{w}(\lambda)\right) - u\left(\underline{w}(\lambda) - x\right).$$

It is now easy to see that (4) and (5) jointly imply that (2) and (3) are satisfied, and we are done. $\qquad\square$

We remark in passing that the complementarity assumption on v is not needed for Proposition 1.

At the same time, the model also admits purely segregated equilibria. Such equilibria are described by a different set of conditions. We study, first, what might be called *single-cut* equilibria, in which I is divided into two intervals, with skilled individuals on (say) the right, and unskilled individuals on the left.

To describe single-cut equilibria, define for any c in the interior of I, the closed intervals $R(c)$ and $L(c)$ to the right and left of c in the obvious way. See Figure 3 for an illustration. Now suppose that all individuals in the relative interior of $R(c)$ are skilled, and all those in the relative interior of $L(c)$ are unskilled. (That leaves open just the measure-0 issue of what households exactly at c do, something we'll settle later.) Define a function ρ on I by

$$(6) \qquad \rho(c) \equiv \frac{F(N^+)}{F(N)},$$

where $N^+ \equiv [c, c + \epsilon]$ and $N = [c - \epsilon, c + \epsilon]$. Because $f(c) > 0$ in the interior of F, ρ is well-defined.
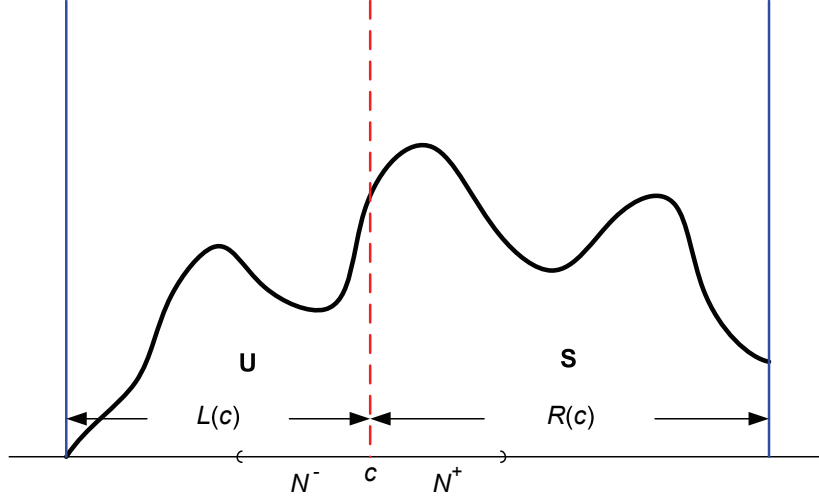
FIGURE 3. A SINGLE-CUT EQUILIBRIUM

Given our spatial arrangement of skilled and unskilled labor, it is easy enough to see that $\rho(c)$ may be interpreted as the proportion of skilled individuals perceived by c, provided that the cut lies at c. Now individuals at location j just to the right of c see a skill proportion that converges to $\rho(c)$ as $j \to c$, so by a trivial continuity argument, a necessary condition for the cut at c to be a steady state is that

$$u\left(\bar{w}(\lambda(c))\right) - u\left(\bar{w}(\lambda(c)) - x\right) \leq \Psi\left(\lambda(c), a(c)\right),$$

where $\lambda(c) = F(R(c))$ is the aggregate proportion of skilled labor generated by the single cut at c, and

$$(7) \qquad a(c) = \rho(c)\bar{w}\left(\lambda(c)\right) + [1 - \rho(c)]\underline{w}\left(\lambda(c)\right).$$

Similarly, using unskilled individuals just to the left of c, we must conclude that a second necessary condition for the cut at c to be a steady state is

$$u\left(\underline{w}(\lambda(c))\right) - u\left(\underline{w}(\lambda(c)) - x\right) \geq \Psi\left(\lambda(c), a(c)\right).$$

Combining these two inequalities, we obtain a necessary condition for the cut at c to generate a steady state:

$$(8) \qquad u\left(\bar{w}(\lambda(c))\right) - u\left(\bar{w}(\lambda(c)) - x\right) \leq \Psi\left(\lambda(c), a(c)\right) \leq u\left(\underline{w}(\lambda(c))\right) - u\left(\underline{w}(\lambda(c)) - x\right).$$

(Now assign households at c to be skilled or unskilled depending on which of these inequalities hold strictly. If both hold with equality, it doesn't matter.)

By the complementarity condition, (7) and (8) must be *sufficient* as well. The reason is that as we move away from c to the right (resp. left), the proportion of skilled people must rise (resp. fall), so that aspirations rise (resp. fall). If incentives are correct at the cut they must therefore be correct in the interior. We conclude that any cut that satisfies (7) must generate a segregated steady state.

Proposition **2.** *A segregated equilibrium exists.*

Proof. Define $\zeta(c) \equiv \Psi(\lambda(c), a(c))$. It is easy to see that ζ is continuous.

Let I be the interval $[\underline{\iota}, \bar{\iota}]$. Note that as $c \uparrow \bar{\iota}$, $\lambda(c) \downarrow 0$, so that $\bar{w}(\lambda(c)) \uparrow \infty$ and $\underline{w}(\lambda(c)) \downarrow 0$. By complementarity, we see that

$$\zeta(c) \equiv \Psi(\lambda(c), a(c)) \geq \Psi(\lambda(c), 0) \to \infty.$$

Now define i^* by the condition that $\lambda(i^*) = \bar{\lambda}$. Again, it is easy to see that as $c \downarrow i^*$, $\zeta(c) \to 0$. At the same time, $u(\bar{w}(\lambda(c))) - u(\bar{w}(\lambda(c)) - x)$ is bounded, continuous and positive on $[i^*, \bar{\iota}]$.

We must therefore conclude that there exists $c \in (i^*, \bar{\iota})$ such that

(9) $$u(\bar{w}(\lambda(c))) - u(\bar{w}(\lambda(c)) - x) = \zeta(c).$$

Because u is concave and $c \geq i^*$, we know that

(10) $$u(\bar{w}(\lambda(c))) - u(\bar{w}(\lambda(c)) - x) \leq u\left(\underline{w}(\lambda(c))\right) - u\left(\underline{w}(\lambda(c)) - x\right).$$

It is now easy to see that (9) and (10) jointly imply that (7) and (8) are satisfied, and we are done.

□

4. Equilibrium Patterns for Linear Utility

Steady state equilibria are driven by three features of the model. First, there is the nature of aspirations and how they affect the incentive to educate a child. This is summarized in the function v. Second, there is the general equilibrium effect: the fact that aggregate skill ratios affect wages. These two features lead to a particular theory of social interactions which are mediated by market prices. Finally, skilled dynasties have different steady state wealths and therefore different (utility) costs of educating their children. These varying costs are reflected in the steady-state equilibrium inequalities such as (3) and (8).

This last feature is generally a shorthand for imperfect or altogether missing capital markets (see Loury 1981, Ray 1990, and Mookherjee and Ray 2003), and it is well-known that the absence of such markets often leads to a multiplicity of steady states (Banerjee and Newman 1993, Galor and Zeira 1993). This model is no exception. For instance, whenever the inequality in (3) holds strictly for some λ, there is a continuum of unsegregated steady states, and whenever the inequality in (8) holds strictly for some $c \in I$, there is a continuum of single-cut segregated steady states. And a slight modification of the proofs of Propositions 1 and 2 tell us that these strict inequalities will indeed hold (for some λ and some c), provided that the direct utility function u is *strictly* concave.[7]

The previous section shows that there is no *formal* problem in integrating all three features — social interaction via aspirations, general equilibrium, and missing capital markets — into a single model. Yet, as is often the case when a model includes several different components, it becomes harder to go deeper into particular predictions. Because our main interest in this

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[7]Mookherjee and Ray (2003) prove that with a rich enough set of occupational choices, this multiplicity disappears. However, we do not incorporate more than two occupations in the current model.

paper concerns the interaction of aspirations with equilibrium prices, we specialize in this section to the case of a linear direct utility function u. We assume that $u(z) = z$, and therefore deliberately depart from the imperfect-capital-markets literature cited above.

This is not to say that missing capital markets play no role. It may well be that the private cost of education x overstates the social cost of education, in part because capital markets are absent or imperfect. However, the linearity assumption means that the resulting distortion is the same for all households irrespective of wealth. In any case, for the positive analysis at least, such an interpretation is unnecessary. (The interpretation will matter, however, when we do the welfare economics of steady states.)

4.1. **Unsegregated Equilibrium.** First consider unsegregated equilibria with linear direct utility. Invoking (3) for this special case, we see that an equilibrium skill ratio λ is fully characterized by the *equality*

$$\Psi(\lambda, a) = x,$$

where a, it will be remembered, equals $\lambda \bar{w}(\lambda) + (1 - \lambda)\underline{w}(\lambda)$, which equals $T(\lambda, 1 - \lambda)$ in turn. As we've already seen, steady states exist, but there may well be many of them, despite the linearity of utility. Suppose that λ_1 is one such steady state. Consider the thought experiment of increasing λ_1 to λ. The direct effect of this is to lower the value of Ψ (this is because the wage differential narrows, lowering the incentive for skill acquisition). At the same time, $T(\lambda, 1 - \lambda)$ may well go up, raising a. By complementarity, this increases the incentive for skill acquisition. The net effect is ambiguous. If, indeed, Ψ goes up as a result, we can be sure that there will exist yet another steady state $\lambda_2 > \lambda > \lambda_1$.[8]

The potential multiplicity here is, however, different from what we observe with strictly concave u. Here, steady states are generically isolated and finite. In the strictly concave case, a continuum of steady states invariably exists.

4.2. **Segregated Equilibrium.** Now we turn to segregated equilibrium. As already discussed, we shall study pure segregated equilibria, in which the length of each segregation interval exceeds ϵ, which is the width of the cognitive window on any one side. This concept makes particular sense when ϵ is small relative to the spread of society as a whole. For instance, pure segregation can never occur if ϵ is so large as to cover the entire interval I from the starting viewpoint of any location in I.

Thus a purely segregated equilibrium generates a finite collection of *cuts*, which we represent by the ordered set $C = \{c_1, \ldots, c_m\} \subset I$. Define $c_0 \equiv \underline{\iota}$ and $c_{m+1} \equiv \bar{\iota}$; then pure segregation implies that $c_{k+1} - c_k > \epsilon$ for all $k = 0, \ldots, m$. (Recall Figure 2.) Moreover, within the consecutive intervals of I generated by the cuts in C, there are alternating zones of skilled and unskilled labor.

One useful implication of pure segregation is that a person located at a cut $c \in C$ sees only skilled people on one side and only unskilled people on the other. Her perceived ratio of skilled individuals will therefore depend entirely on the value $\rho(c)$, where ρ is defined in

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[8]Lemma 1, used in another context, contains a formal statement of this assertion.

(6). More specifically, her perceived ratio will equal $\rho(c)$ if the zone to the right of her is populated by the skilled, and it will equal $1 - \rho(c)$ if the zone to the left of her is populated by the skilled. To write this in a compact way, define a function $\chi(c)$ on C that takes the value 1 if a skilled interval lies to the right of c, and 0 otherwise. Then an individual at c must perceive the local skill ratio

$$\sigma(c) \equiv \chi(c)\rho(c) + [1 - \chi(c)][1 - \rho(c)],$$

so that if the *aggregate* skill ratio is λ, an individual positioned at the cut c must have the aspiration

(11) $$a(c) = \sigma(c)\bar{w}(\lambda) + [1 - \sigma(c)]\underline{w}(\lambda).$$

Now, the same argument that we used for a single-cut equilibrium shows that at *any* cut c in a purely segregated equilibrium with aggregate skill ratio λ,

$$u(\bar{w}(\lambda)) - u(\bar{w}(\lambda) - x) \leq \Psi(\lambda, a(c)) \leq u\left(\underline{w}(\lambda)\right) - u\left(\underline{w}(\lambda) - x\right).$$

Invoking the assumption that u is linear, this inequality reduces to the condition

(12) $$\Psi(\lambda, a(c)) = x.$$

Finally, we pin down λ, which is simply the aggregate mass of all skilled intervals:

(13) $$\lambda = \sum_{k=0}^{m} F([c_k, c_{k+1}])\chi(c_k).$$

By the complementarity assumption, conditions (11)–(13) completely characterize all purely segregated equilibria.

We place one further restriction on purely segregated equilibria: we ask that their cuts c have the property that both $c + \epsilon$ and $c - \epsilon$ lie on the same stretch of f as c does. It is easy to verify that this is a generic property of segregated equilibria provided we require it for all ϵ small enough.[9] We call such equilibria *regular*.

We now state our basic proposition for purely segregated regular equilibria under linear direct utility.

PROPOSITION 3. *In any purely segregated regular equilibrium, each stretch of f can contain at most one cut.*

Proof. Suppose, on the contrary, that there are two consecutive cuts, call them c and c', along some interval of I on which f is strictly monotone. We claim that $\sigma(c) \neq \sigma(c')$. To see this, note that $\chi(c) \neq \chi(c')$, simply because c and c' are consecutive. Therefore $|\sigma(c) - \sigma(c')| = |\rho(c) + \rho(c') - 1|$. However, because the equilibrium is regular, along the same

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[9]Suppose that there is a sequence of ϵ-windows converging to zero and a corresponding sequence of purely segregated equilibria with a nonregular cut in each of them. Then, using the fact that each cut must contain an indifferent person, who must therefore collectively have the same aspirations as one another, it is possible to show that *all* cuts converge to local peaks or troughs as $\epsilon \downarrow 0$. This means, in turn, that aggregate λ can have only one of a finite possible set of values (use (13)), and it also means that limit aspirations for indifferent individuals have at most a finite number of values as well. Combining, we see that $\Psi(\lambda, a(c))$ converges to one of a finite number of possible values as ϵ vanishes, and therefore equation (12) will generically fail.
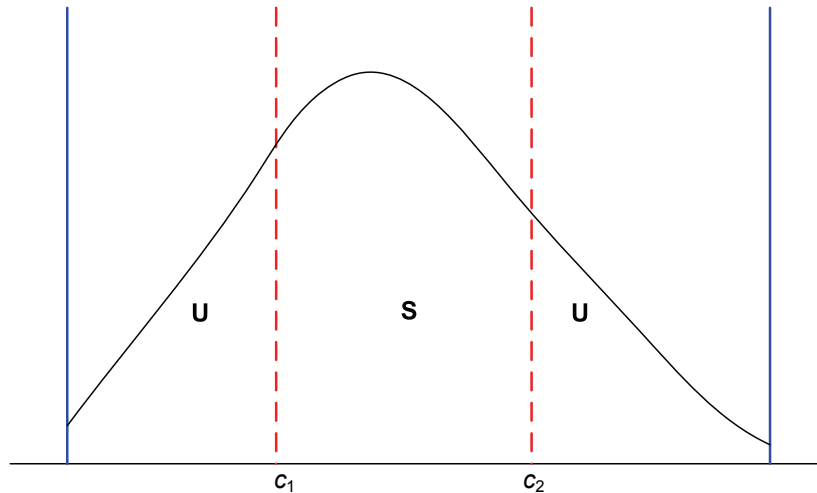
FIGURE 4. A Two-Cut Equilibrium with Skilled Labor Occupying the Mode

stretch either $\rho(c)$ and $\rho(c')$ are both greater than 1/2, or they are both less than 1/2. This proves the claim.

Because $\bar{w}(\lambda) > \underline{w}(\lambda)$ in any equilibrium, we must conclude from (11) that $a(c) \neq a(c')$. Therefore, by the claim, at least one of c or c' must fail (12), which shows that both cannot be equilibrium cuts. This is a contradiction. □

Proposition 3 has strong implications for special cases of the model. For instance, it severely restricts equilibrium outcomes when the distribution of population across locations is unimodal.

COROLLARY 1. *If f is unimodal, then a purely segregated regular equilibrium can involve at most two cutoffs, and if there are two, they must be on either side of the mode.*

Proof. A unimodal f has no more than two stretches. Apply Proposition 3. □

The restriction depends on the assumption that f is nonconstant almost everywhere. It is easy to see that Corollary 1 is false when f is uniform on I: in that case there are purely segregated equilibria with one, two, or several cuts (provided that ϵ is small enough), and no particular spatial pattern of segregation emerges. In the "strictly" unimodal case, however, we see that purely segregated equilibria must assume a very simple form. Moreover, in the two-cut case one of the two skill groups must occupy the highly populated "center zone" around the mode, while the other skill group is banished to the low-density "periphery". Figure 4 illustrates a two-cut equilibrium in the unimodal case.

A similar pattern obtains for multimodal distributions:

COROLLARY 2. *If f is multimodal with n local modes, then a purely segregated regular equilibrium involves at most $2n$ cutoffs, and consecutive cutoffs must be located on stretches of f with slopes of opposite sign.*

Proof. An f with n local modes has no more than $2n$ stretches. Apply Proposition 3. □

The fact that consecutive cutoffs must be located on stretches of f with slopes of opposite sign allows us to generalize the center-periphery pattern in the unimodal case. Suppose that a multi-cut equilibrium begins with a segment of skilled individuals, and exhibits its first cut on a downward stretch. Then there must be a local mode to the left of the cut (a "local center") occupied by S. But this one fact now *necessitates* that every succeeding unskilled segment must wrap around at least one trough (a "local periphery"), and moreover, that every succeeding skilled area again wrap around around at least one more local center. On the other hand, if the first cut occurs on an upward stretch, then every unskilled zone must contain at least one local center, and every skilled zone must contain at least one local periphery.

The structure of local interactions, coupled with the market-clearing nature of prices, jointly impose this global structure on the spatial outcomes. In particular, the fact that prices clear markets, together with the assumption of linear direct utility, allow us to infer the existence of an "indifferent" agent, for whom aspirations and skill premia exactly balance out so that the acquisition of skills is an exact toss-up.[10] This indifferent agent imposes a lot of structure on spatial outcomes. The central idea used to obtain this structure is the fact that an indifferent agent in two different locations must locally see the same mix of skilled and unskilled individuals. (In the particular model we study here, this means that two neighboring cuts must lie on stretches of f with opposite slope.)

4.3. **Existence of Multi-Cut Equilibria.** It is important to note that (unlike single-cut equilibria), multi-cut purely segregated equilibria do not always exist. There is a good reason for this: the same structural discipline that pins down the spatial patterns of an equilibrium may on occasion (but not always!) be too restrictive to permit existence. To see this, imagine a unimodal distribution in which the upward stretch has markedly different slope from the downward stretch. Now the indifferent agents on either side of a two-cut equilibrium must have similar local perceptions; i.e., they must see the same mix of skilled and unskilled individuals in the ϵ-window around them. In other words, their locations must be chosen so that the local slopes of f around them (suitably weighted by height) are the same. However, the phrase "markedly different slopes" above means that in order to achieve the desired equality, we lose flexibility over the *aggregate* proportions of skilled to unskilled labor. In short, the proportions needed to assure equality of local skill ratios may be inconsistent with those needed to create market-clearing at the macroeconomic level.

This argument is informal and imprecise, but the following example tells us that it works.

EXAMPLE 1. *Let* $I \equiv [\underline{\iota}, \bar{\iota}] = \left[-1/\alpha, \, -1/\gamma \cdot \ln\left(1 - \gamma(1 - \frac{1}{2\alpha})\right) \right]$ *for* $\alpha \gg 1$, *and* $\gamma > 0$ *sufficiently small so that* $\alpha \geq 4\gamma$ *and* $\bar{\iota}$ *is large compared to* $1/\alpha$. *Consider the unimodal density function*

$$f(i) = \begin{cases} 1 + \alpha i, & i < 0 \\ e^{-\gamma i}, & i \geq 0. \end{cases}$$

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[10]The existence of indifferent agents also presumes that the steady state is interior in skill acquisition. The Inada condition guarantees that skill premia are extremely large if no one acquires skills. Likewise, if everyone acquires skills, all skill premia vanish. These two assertions guarantee interiority.

On the linearly increasing part of f, $\rho(i)$ falls monotonically from $\rho(\underline{\iota}) = 1$ to

$$\rho(0) = \frac{F([0, \epsilon])}{F([-\epsilon, 0]) + F([0, \epsilon])}.$$

Over the exponentially decreasing stretch, ρ falls continuously from $\rho(0)$ to $\rho(\epsilon) = (1 - e^{\epsilon})/(e^{-\epsilon} - e^{\epsilon})$, stays constant at this level from ϵ up to $\bar{\iota} - \epsilon$, and then drops continuously again to $\rho(\bar{\iota}) = 0$. Cuts at any c and $c' \geq c + \epsilon$ must satisfy $\rho(c) = 1 - \rho(c')$ in order to involve identical aspirations. So if $\rho(c) > 1 - \rho(\epsilon)$ for every $c \in [\underline{\iota}, 0]$, then c' necessarily must be located towards the very right, namely, in the interval $[\bar{\iota} - \epsilon, \bar{\iota}]$; and the corresponding aggregate skill ratio is either at least $\lambda = \bar{\iota} - \epsilon$ (agents left of c are unskilled) or at most $\lambda' = 1 - \bar{\iota} + \epsilon$ (agents left of c are skilled). A segregated equilibrium can then only exist if the given combination of technology T and preferences v admits such high or low skill ratios; many combinations do not, and for these no segregated equilibrium can exist.

It remains to confirm that indeed $\rho(c) > 1 - \rho(\epsilon)$ for every $c \in [\underline{\iota}, 0]$ given the above density f. Since $\rho(i)$ is strictly decreasing for $i < 0$, it suffices to check that

$$\rho(0) = \frac{F([0, \epsilon])}{F([-\epsilon, 0]) + F([0, \epsilon])} > \frac{F([0, \epsilon])}{F([0, \epsilon]) + F([\epsilon, 2\epsilon])} = 1 - \rho(\epsilon),$$

or

$$F([-\epsilon, 0]) = \epsilon(1 - \alpha\epsilon/2) < F([\epsilon, 2\epsilon]) = [e^{-\gamma\epsilon} - e^{-2\gamma\epsilon}]/\gamma.$$

Because $e^{-\gamma i}$ is convex,

$$-\frac{\epsilon}{\gamma} \left. \frac{\partial e^{-\gamma i}}{\partial i} \right|_{i=2\epsilon} = \epsilon e^{-2\gamma\epsilon} = \epsilon \left(1 + \frac{-2\gamma\epsilon}{1!} + \frac{(-2\gamma\epsilon)^2}{2!} + \frac{(-2\gamma\epsilon)^3}{3!} + \dots \right) < F([\epsilon, 2\epsilon]).$$

Given that $|-2\gamma\epsilon| < 1$ for small ϵ, a sufficient condition for $\rho(0) > 1 - \rho(\epsilon)$ is therefore

$$1 - \alpha\epsilon/2 \leq 1 - 2\gamma\epsilon$$

or simply $\alpha \geq 4\gamma$, as we have assumed above.

That said, it is possible to provide sufficient conditions for the existence of multi-cut equilibria. We have, for instance:

PROPOSITION 4. *Suppose that f is unimodal and symmetric. Then a purely segregated regular two-cut equilibrium exists, provided ϵ is small enough.*

Proof. We prove existence of a two-cut equilibrium with the unskilled in the center and the skilled on the two peripheries. Without loss of generality the support of f is $[0, 1]$ and the unique mode is located at $1/2$. For any cut $c \in [1/2, 1]$ it is natural to imagine its "mirror cut" on $[0, 1/2]$, which is the point $1 - c$. Remembering that skilled labor is being placed on the sides, we may define

$$\lambda(c) \equiv F([c, 1]) + F([0, 1 - c])$$

to be the aggregate amount of skilled labor associated with c. It is obvious that there exists $\underline{c} > 1/2$ such that $\lambda(\underline{c}) = \bar{\lambda}$, at which point wages are equalized. Clearly, an equilibrium c must exceed $\underline{c}$. Let B be the set $[\underline{c}, 1]$. Define $\underline{\epsilon} \equiv \underline{c} - 1/2$, and consider any $\epsilon < \underline{\epsilon}$. Notice that any equilibrium with cuts $\{1 - c, c\}$, where $c \in B$, must be purely segregated and regular.

It remains to show that there is such an equilibrium. To do so, define

$$a(c) \equiv \rho(c)\bar{w}\left(\lambda(c)\right) + [1 - \rho(c)]\underline{w}\left(\lambda(c)\right).$$

and note that it is continuous on B. Moreover,

$$\Psi\left(\lambda(\underline{c}), a(\underline{c})\right) < x,$$

simply because there is no wage differential when $c = \underline{c}$, while

$$\Psi\left(\lambda(c), a(c)\right) > x \text{ for all } c \text{ close to } 1,$$

because $\lambda(c) \downarrow 0$ as $c \uparrow 1$, so that $\bar{w}\left(\lambda(c)\right) \uparrow \infty$.

It follows that there exists $c \in B$ such that $\Psi\left(\lambda(c), a(c)\right) = x$. It is now easy to verify that $\{1 - c, c\}$ forms a double-cut equilibrium. ☐

4.4. **Comparisons Across Equilibria.** Our model also makes possible sharp comparisons across equilibria. Without dwelling on the welfare economics of equilibrium outcomes (for the moment), we can still compare equilibria in terms of their macroeconomic propensities to generate skill. There are different comparisons possible.

4.4.1. *City-Skilled Versus City-Unskilled Equilibria.* First we look at purely segregated multi-cut equilibria. Recall from the discussion following Corollary 2 that we've unearthed a particular spatial pattern of such equilibria. We begin by tightening this discussion. Define a segregated equilibrium to be *city-skilled* if there is some equilibrium cut which divides a skilled local mode from an unskilled local trough. If, on the other hand, there is some equilibrium cut that divides an unskilled local mode from a skilled local trough, call the equilibrium *city-unskilled*.

Note that these definitions refer to local properties (of *some* equilibrium cut in the whole set of cuts) so that in principle a segregated equilibrium could be both city-skilled *and* city-unskilled. But this cannot happen:

PROPOSITION 5. *A purely segregated regular equilibrium must be city-skilled or city-unskilled, and it can never be both.*

Proof. If a purely segregated equilibrium is neither city-skilled nor city-unskilled, *all* its cuts must lie on local peaks and troughs, which regularity rules out.

Suppose a purely segregated regular equilibrium is both city-skilled (the cut c verifies this) and city-unskilled (the cut c' verifies this). At c, it must be the case that $\sigma(c) > 1/2$, because individuals at c must see more skilled individuals than unskilled (we use regularity here). The opposite is true at c'. But then individuals at c and c' cannot have the same aspirations, which means that they cannot both be indifferent, a contradiction. ☐

Can city-skilled and city-unskilled equilibria coexist in the same model? There is no reason why not. For instance, we have:

PROPOSITION 6. *Suppose that f is unimodal and symmetric. Then both city-skilled and city-unskilled two-cut equilibria exist, provided ϵ is small enough.*
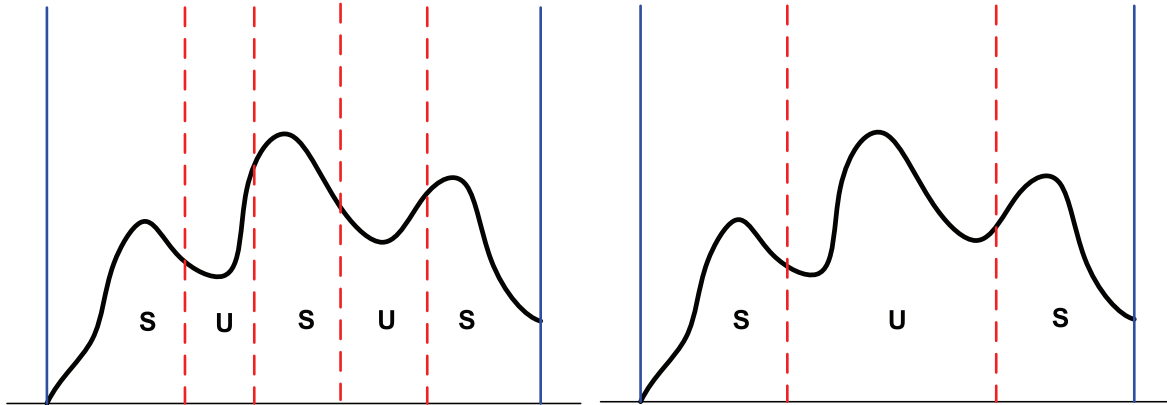
FIGURE 5. TWO CITY-SKILLED EQUILIBRIA

The proof of this proposition follows exactly the same lines as Proposition 4, and is therefore omitted.

What do city-skilled equilibria look like? By definition, *some* local mode (a "city") is occupied by skilled individuals. But this necessitates that *every* skilled interval must contain a local mode: in every skilled segment of a city-skilled equilibrium, there is a city. Exactly the opposite is true of city-unskilled equilibrium: every unskilled segment must contain a local city. In short, a city-unskilled equilibria exhibit "inner cities" in every interval of unskilled labor.

We add, however, that city-skilled equilibria don't rule out unskilled inner cities, nor do city-unskilled equilibria exclude the possibility of skilled urban areas. Figure 5 describes two city-skilled equilibria. In the first, there are no unskilled households around a local mode. In the second, there are unskilled local modes.

Of some interest is the fact that we can compare city-skilled and city-unskilled equilibria in terms of the aggregate skills they generate.

PROPOSITION **7.** *A city-skilled equilibrium must generate more skilled labor in the aggregate than any city-unskilled equilibrium.*

Proof. Consider a city-skilled equilibrium with aggregate skills λ, and consider a cut c that separates a skilled local mode from an unskilled local trough. It is easy to see that $\sigma(c) > 1/2$. In a similar way, there is a cut c' for a city-unskilled equilibrium (displaying aggregate skills λ') with $\sigma(c') < 1/2$. Now suppose, contrary to our assertion, that $\lambda \leq \lambda'$. Then $\bar{w}(\lambda) \geq \bar{w}(\lambda')$ and $\underline{w}(\lambda) \leq \underline{w}(\lambda')$, so that if a and a' are the aspirations at cuts c and c' under the two equilibria,

$$a = \sigma(c)\bar{w}(\lambda) + [1 - \sigma(c)]\underline{w}(\lambda) > \sigma(c')\bar{w}(\lambda') + [1 - \sigma(c')]\underline{w}(\lambda') = a'.$$

By complementarity of v and the wage and aspiration comparisons above, we must conclude that

$$\Psi(\lambda, a) > \Psi(\lambda', a'),$$

which contradicts condition (12) for at least one of the presumed equilibria. □

4.4.2. *Segregated Versus Unsegregated Equilibria.* Unsegregated and segregated equilibria can also be compared. The comparison is not unambiguous, however. We conduct the analysis for small window sizes. The following proposition fully describes the limit outcomes of pure segregated equilibria as the window length ϵ shrinks to zero.

PROPOSITION 8. *Let ϵ converge to 0, and $C(\epsilon)$ be a corresponding sequence of purely segregated equilibrium cut sets. If $\lambda(\epsilon)$ is the aggregate skill generated by $C(\epsilon)$, then $\lambda \to \lambda^*$, where λ^* solves*

$$(14) \qquad \Psi\left(\lambda, \frac{\bar{w}(\lambda) + \underline{w}(\lambda)}{2}\right) = x.$$

Proof. First we claim that there exists a selection $c(\epsilon) \in C(\epsilon)$ for every ϵ, such that $c(\epsilon) \to c^*$, with $\iota < c^* < \bar{\iota}$. If this were false, then all such selections have limit points that are either ι or $\bar{\iota}$. It is easy to see that such a property implies the convergence of $\lambda(\epsilon)$ to either 0 or 1. Now the equilibrium λ can never exceed $\bar{\lambda}$ (which solves $\bar{w}(\bar{\lambda}) = \underline{w}(\bar{\lambda})$), which means that $\lambda(\epsilon) \to 0$. But then for small enough ϵ, we see that for every household at i with equilibrium aspiration $a_i(\epsilon)$,

$$\Psi\left(\lambda(\epsilon), a_i(\epsilon)\right) \geq \Psi\left(\lambda(\epsilon), 0\right) > x,$$

because v is unbounded in w by assumption. That is, all households want to acquire skills, which contradicts the presumption that equilibrium $\lambda(\epsilon)$ is close to 0. This proves the claim.

Pick a selection $c(\epsilon)$ as given by the claim, with $c(\epsilon) \to c^* \in (\iota, \bar{\iota})$ as $\epsilon \to 0$. It is easy to see that for all ϵ small enough, both $c(\epsilon) - \epsilon$ and $c(\epsilon) + \epsilon$ are in the interior of I. Define $\rho(\epsilon) \equiv \rho(c(\epsilon))$. We claim that $\rho(\epsilon) \to 1/2$. Recalling (6), we see that

$$\rho(\epsilon) = \frac{F([c(\epsilon), c(\epsilon) + \epsilon])}{F([c(\epsilon) - \epsilon, c(\epsilon) + \epsilon])} = \frac{\int_{c(\epsilon)}^{c(\epsilon)+\epsilon} f(i)di}{\int_{c(\epsilon)-\epsilon}^{c(\epsilon)+\epsilon} f(i)di}.$$

Because f is continuous and $f(c^*) > 0$, the claim is proved. As a trivial corollary of this claim, $\sigma(c(\epsilon)) \to 1/2$ as well.

To complete the proof, note that by indifference at the equilibrium cut $c(\epsilon)$, we have

$$\Psi\left(\lambda(\epsilon), a(\epsilon)\right) = x \text{ for all } \epsilon,$$

where $a(\epsilon)$ denotes aspirations at the cut $c(\epsilon)$, given by

$$a(\epsilon) = \sigma(c(\epsilon))\bar{w}\left(\lambda(\epsilon)\right) + [1 - \sigma(c(\epsilon))]\underline{w}\left(\lambda(\epsilon)\right).$$

Combine these last two equations with the fact that $\sigma(\epsilon) \to 1/2$ as $\epsilon \to 0$, and pass to the limit to obtain the desired result. $\qquad \square$

The proposition states that as the window size ϵ becomes vanishingly small, *all* purely segregated equilibria, no matter what their spatial structure, must generate the same aggregate quantity of skills. The intuition is very simple: as the window size becomes small, an indifferent individual placed at a cut sees approximately equal numbers of skilled and unskilled individuals, no matter what the density function at her location looks like.[11] The

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[11] This observation is similar to a parallel step used in the theory of global games, in which the rank-order of a particular signal is uniformly distributed as the noise becomes small.

incentives of the indifferent individual(s) must pin down the equilibrium wage differential; hence we obtain (14) for *all* purely segregated equilibria as ϵ becomes vanishingly small.

How literally we take this result depends in large part on our intuition about cognitive windows. We are agnostic on this question. We believe, in line with a large literature on local interactions, that the case of small windows is of interest, but at the same time do not push the line that such windows must be *vanishingly* small relative to the economy as a whole. (If we did, we would not have reported the comparisons in the previous section: there would be nothing to compare.)

In the end, we view the case of vanishingly small windows as a convenient device to compare segregated versus unsegregated equilibria, to which we now turn. How compelling the following observations are to the reader will depend, in part, on how comfortable she is with extremely small window sizes.

Proposition 8 holds a simple clue to comparing unsegregated and segregated equilibria. To exploit this, we parameterize the skill bias of the technology by some parameter A. Write the production function as some $T(\lambda, 1 - \lambda, A)$, where A is normalized to lie in $[0, 1]$. Think of higher values of A as indexing production functions with greater degrees of skill bias.

Write the skilled and unskilled wages as functions $\bar{w}(\lambda, A)$ and $\underline{w}(\lambda, A)$ and assume these are continuous in A. We will need the following minimal restrictions, placed on the 50-50 input configuration $\lambda = 1/2$.

[A.1] As $A \to 1$, $\underline{w}(1/2, A)$ converges to 0, and for small enough A, $\bar{w}(1/2, A) \leq \underline{w}(1/2, A)$.[12]

[A.2] As $A \to 1$, the skilled wage increases enough (relative to the unskilled wage, which is converging to 0), to make investment worthwhile even at zero aspirations. That is, $v(\bar{w}(1/2, 1), 0) - v(0, 0) > x$.

The following proposition shows that the skill bias of the technology informs the comparison across segregated and unsegregated equilibrium:

PROPOSITION 9. *Under conditions [A.1] and [A.2], there exist two threshold values of the skill bias, $\bar{A}$ and $\underline{A}$, such that*

1. If $A > \bar{A}$ then there is an unsegregated equilibrium with higher skills than any segregated equilibrium, provided that ϵ is small enough.

2. If $A < \underline{A}$ then for ϵ small enough, every purely segregated equilibrium generates higher skills than any unsegregated equilibrium.

Proof. We prove statement 1. The proof of statement 2 essentially reverses the argument below (we indicate the main steps at the end).

Include A explicitly in the various expressions below to indicate that the corresponding variables depend on this parameter. For instance, we define $\Psi(\lambda, a, A) \equiv v(\bar{w}(\lambda, A), a) - v\big(\underline{w}(\lambda, A), a\big)$.

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[12] A more symmetric way of writing this is to say that $\bar{w}(1/2, A)$ converges to 0 as $A \to 0$. But the weaker form that we adopt allows for the possibility that skilled labor can do unskilled jobs; see footnote 5.

LEMMA **1.** *Fix A. Suppose that for some λ_0 the following is true:*

$$\Psi\left(\lambda_0, a(\lambda_0, A), A\right) > x,$$

where for all λ, we define

$$a(\lambda, A) \equiv \lambda \bar{w}(\lambda, A) + (1 - \lambda)\underline{w}(\lambda, A).$$

Then there exists an unsegregated equilibrium with aggregate skill $\lambda > \lambda_0$.

Proof. Certainly, it must be the case that $\lambda_0 < \bar{\lambda}(A)$, for skilled and unskilled wages are equalized at the latter value, and so $\Psi\left(\lambda', a(\lambda', A), A\right) < x$ for all $\lambda' \geq \bar{\lambda}(A)$. By the intermediate value theorem, there exists $\lambda > \lambda_0$ such that $\Psi\left(\lambda, a(\lambda, A), A\right) = x$, and this must be an unsegregated equilibrium. □

By [A.1] and [A.2], there exists a threshold value $\bar{A}$ such that

$$v\left(\bar{w}(1/2, A), 0\right) - v\left(\underline{w}(1/2, A), 0\right) - x > 0 \text{ for all } A > \bar{A}.$$

By complementarity, it must therefore be the case that

(15) $\qquad \Psi\left(1/2, a(1/2, A), A\right) - x = v\left(\bar{w}(1/2, A), a(1/2, A)\right) - v\left(\underline{w}(1/2, A), a(1/2, A)\right) - x > 0$

for all $A > \bar{A}$, where remember that

(16) $$a(1/2, A) = \frac{1}{2}\left[\bar{w}(1/2, A) + \underline{w}(1/2, A)\right].$$

The relationships (15) and (16) tell us that $\lambda_0 = 1/2$ satisfies the conditions of Lemma 1. We therefore conclude that for each $A > \bar{A}$, there exists an unsegregated equilibrium with $\lambda > 1/2$.

Fix any such A. Proposition 8 tells us that if λ^* is a limit (as $\epsilon \to 0$) of some sequence of purely segregated equilibria, then

(17) $$\Psi\left(\lambda^*, \frac{\bar{w}(\lambda^*) + \underline{w}(\lambda^*)}{2}, A\right) = x.$$

If $\lambda^* \leq 1/2$ the proof is complete, because the unsegregated $\lambda > 1/2$. So $\lambda^* > 1/2$. But then, by complementarity,

$$\Psi\left(\lambda^*, \lambda^* \bar{w}(\lambda^*) + [1 - \lambda^*]\underline{w}(\lambda^*), A\right) > \Psi\left(\lambda^*, \frac{\bar{w}(\lambda^*) + \underline{w}(\lambda^*)}{2}, A\right),$$

and combining this inequality with (17), we must conclude that

$$\Psi\left(\lambda^*, \lambda^* \bar{w}(\lambda^*) + [1 - \lambda^*]\underline{w}(\lambda^*), A\right) > x.$$

This time $\lambda_0 = \lambda^*$ satisfies the conditions of Lemma 1. So there exists an unsegregated λ that strictly exceeds λ^*, and the proof of statement 1 is complete.

To prove statement 2, first note that Lemma 1 is also true when both inequalities are reversed. Next, using [A.1], we show the existence of a lower threshold $\underline{A}$ such that the reverse inequality holds in (15). Now follow the same lines as in the rest of the proof, reversing the inequalities where needed. □

The proposition suggests that in societies which depend heavily on skilled labor in production, there is a case for desegregation on the grounds of greater skill accumulation in the aggregate. Desegregation lowers the aspirational incentives of those in already-skilled neighborhoods, but raises incentives for the unskilled who come into contact with them. The net outcome, however, is positive provided that society is skill-intensive in the first place.

A perusal of the proof will reveal that the salubrious effects of desegregation are generally higher in situations in which the majority of the population are skilled in unsegregated equilibrium.

Yet the opposite is true in poorer societies where the paucity of physical capital (not explicitly modeled here) lowers the relative demand for skilled labor. Proposition 9 then suggests that segregation may be a better generator of skills. In line with the first part of the proposition, such a situation is likely to be associated with one in which a minority of the population is skilled in unsegregated equilibrium.

These results are to be treated with a certain degree of caution. More work needs to be done to see how they will hold up in more general specifications. We suspect, though, that the main arguments will go through. For instance, one might allow for two components of aspirations: one "local" in the sense developed in this paper, and the other "global", related to per-capita income economy-wide. It is not difficult to see that in this case, the ordering of Proposition 9 would be preserved and even magnified in intensity. Because per-capita income is positively related to skill acquisition in the relevant range of equilibrium outcomes, a global component of aspirations would simply reinforce the advantages of segregation or desegregation (depending on which parameters of that proposition are in force).

5. Welfare Economics

Two considerations are fundamental to any discussion of welfare in these models. First, we need to fix ideas on whether equilibrium skill aggregates are too small or large relative to some "first best" steady state.[13] Nothing in the logical structure of our model allows us to *necessarily* assert that we have over- or under-accumulation of skills, though the model is entirely compatible with the realistic scenario of under-accumulation. We therefore separately impose the assumption (already made implicitly in the previous section) that at any equilibrium outcome, more skills are socially desirable. It is easy enough to derive such an outcome if we assume (among other things) that the private cost of education (x) outstrips the social cost, perhaps because of imperfections in the capital market.

Next, there is the question of what the v-function looks like. Our characterization of the spatial structure of equilibria was driven entirely by the complementarity of investment incentive. The precise nature of this complementarity does not matter — neither for the existence of segregated and unsegregated equilibria, nor their levels of aggregate human capital investment. However, for welfare it *does* matter whether we see v as increasing or

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[13]There is also the more subtle issue of dynamics. It is entirely possible that a steady state is Pareto-dominated by another steady state, while at the same time the former is Pareto-optimal. (After all, two different steady states display two different initial conditions.) Mookherjee and Ray (2003) contains a more detailed discussion of this issue, which we avoid here.

decreasing in aspirations. Our default assumption would be that v decreases in a: having higher aspirations to start with renders any given achievement less attractive. But the opposite may well be true. For instance, one could (perhaps with some effort!) think of aspirations as a local public good, which brings non-pecuniary benefits: identification with the local community, the inculcation of a work ethic, the setting of healthy goals, and so on.[14]

In what follows, we assume throughout that greater skill generation is a good thing, but we discuss both positive and negative effects of higher aspirations on individual welfare.

5.1. **Altruism as Pure Aspiration.** As a first benchmark, suppose that parental concern for the pecuniary success of their children is driven by parental aspirations alone. That is, having a child with a wage above the local average is considered a success (an achievement parents may brag about), while a wage below the neighbors' average is viewed as a failure, or at least reduces parental well-being. In particular, suppose that meeting exactly the given aspiration level neither enhances nor reduces a parent's satisfaction. A child's human capital thus has the character of a positional good for the parent, with no intrinsic value.[15]

A straightforward example of purely aspirational altruism is one in which v depends *only* on the difference between wage and aspiration. More generally, assume that $v(w, w)$ is constant (say 0). The assumption that v is increasing in its first argument then implies that $v(w, a)$ is decreasing in a: the greater the average achievement of an individual's peers, the less will that individual value any given achievement by her child. Assume, moreover, that v is strictly concave in its first argument. This implies that the aggregate utility associated with aspirational altruism is negative and bounded away from zero in an unsegregated equilibrium with skill ratio λ^u:

$$\lambda^u v(\bar{w}, a) + (1 - \lambda^u) v(\underline{w}, a) < v(a, a) = 0$$

where $a = \lambda^u \bar{w} + (1 - \lambda^u) \underline{w}$.

The aggregate aspirational utility in any segregated equilibrium λ^s, in contrast, approaches zero as $\epsilon \to 0$: the size of the transition zones between skilled and unskilled stretches becomes negligible and households will exactly match their aspirations inside any stretch.

Tack on to this the second set of conditions in Proposition 9, in which the economy has an unskilled-labor bias and segregation generates higher aggregate skills. Under these conditions, total welfare is unambiguously higher in a segregated equilibrium. Segregation entails greater human capital and net output, and it also creates a smaller welfare loss associated with the individual mismatch of wage and aspirations.

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[14]This would come close to the idea that a has a beneficial effect on the individual *cost* of education due to local spillovers caused, for instance, by local school financing or parental involvement at school, role models, or peer effects (see Bénabou 1993; 1996; or Durlauf 1994; 1996, and the empirical references therein). But it is not the same thing. Such cost externalities directly affect only those agents who are, in fact, investing in their child's human capital. In contrast, we are discussing the effect of aspirations on both investors and noninvestors.

[15]A variation of our model might dispense with the OLG setup: the investment decision could refer to an indivisible good, say, a swanky car, whose value is only positional and decreasing in the local share of swanky-car owners, and whose price is determined in an economy-wide market. The spatial ownership patterns are likely to mimic our equilibria, but the welfare implications would probably be different, unless one associates the same kind of macroeconomic productivity effects with car ownership as we do with human capital.

If, on the other hand, the first set of conditions in Proposition 9 applies, the welfare ranking of equilibria is ambiguous: the greater net output under desegregation needs to be traded off against the global "simmering" of unskilled households with unmet aspirations.

To be sure, this sort of analysis presumes that segregated societies can't see beyond their own segregation. That may be an incorrect point of view, though we need to be careful in taking a more nuanced position. One counter-claim is that there is no such thing as segregation: the entire society determines aspirations and therefore investment. We doubt that this is the case. A second, more subtle position is that local aspirations drive investment while global comparisons have no functional role but create resentments for those at the bottom. We don't know if either of these positions is correct but if they are, the welfare pronouncements of this section may need to be revisited.

5.2. **A Direct Concern for Descendant Incomes.** The somewhat disturbing argument in favor of segregation (in the case in which skills are in low demand) is driven by another important assumption. This is the presumption that all education is purely positional: v decreases in a, and any wage increase is entirely neutralized by a corresponding increase in aspirations. It is arguably more realistic to suppose that parents also care about the offspring's wage *per se*, not just relative to a. This scenario is easy enough to capture: assume that $\tilde{v}(w) \equiv v(w, w)$ is actually strictly increasing in w. One possibility would be that $v(w, a) = v_1(w - a) + v_2(w)$ where the function v_1 captures purely aspirational altruism as discussed above, and the function v_2 incorporates a direct concern for descendant incomes.

Even if we retain the assumption that v is concave in w, the aggregate utility associated with aspirational altruism in an unsegregated equilibrium may now very well be sizable. Segregation will also be associated with aspiration utility, but (presuming again that the cognitive window is small), the "aspirational utility difference" is now given by

$$\left\{ \lambda^u v(\bar{w}, a) + (1 - \lambda^u) v(\underline{w}, a) \right\} - \left\{ \lambda^s v(\bar{w}, \bar{w}) + (1 - \lambda^s) v(\underline{w}, \underline{w}) \right\}$$

with $a = \lambda^u \bar{w} + (1 - \lambda^u)\underline{w}$, and this can no longer be signed unequivocally. So even in the low-skill case, the welfare comparison between segregated and unsegregated equilibria is ambiguous and depends on one's precise assumptions.

The difference between welfare without segregation and with it can be usefully decomposed by conducting two thought experiments. One is to hypothetically redistribute total incomes inside local neighborhoods such that every household has an equal income, and hence equal aspirations. Comparing the resulting aspirational welfare with that implied by the actual wage distribution inside local neighborhoods provides a measure of the welfare loss associated with *intra-neighborhood inequality*. We can thus decompose the total welfare in an unsegregated equilibrium as follows:

$$W^u(\lambda^u) \equiv \left\{ F(\lambda^u, 1 - \lambda^u) - \lambda^u x + v(a, a) \right\} - \left\{ v(a, a) - \left(\lambda^u v(\bar{w}, a) + (1 - \lambda^u) v(\underline{w}, a) \right) \right\}$$

with $a = \lambda^u \bar{w} + (1 - \lambda^u)\underline{w}$. The first term in brackets captures the "gross efficiency" of skill λ^u: net output plus the aspirational welfare that would result under an equal income distribution. The second term corrects this by accounting for the social cost of intra-neighborhood inequality associated with λ^u.

The second thought experiment is to hypothetically redistribute the aggregate wage of each local community across skilled and unskilled stretches such that there is no inequality between them. Comparing the resulting aspirational welfare to that implied by the actual wage distribution produces a measure of the welfare loss associated with *inter-neighborhood inequality*. Since there are no differences between distinct neighborhoods in an unsegregated equilibrium, the latter involves no such costs of inter-neighborhood inequality. In contrast, a segregated equilibrium involves no intra-neighborhood inequality (hence no associated welfare costs), but there are inter-neighborhood differences. Total welfare in a segregated equilibrium can therefore be understood as the sum of three components: gross efficiency, zero social costs from intra-neighborhood inequality, and positive social costs stemming from inter-neighborhood inequality (again we treat transition zones as negligible and consider a vanishing ϵ):

$$W^s(\lambda^s) \equiv \left\{ F(\lambda^s, 1 - \lambda^s) - \lambda^s x + v(a, a) \right\} - 0 - \left\{ v(a, a) - \left(\lambda^s v(\bar{w}, \bar{w}) + (1 - \lambda^s) v(\underline{w}, \underline{w}) \right) \right\}.$$

So whilst a segregated equilibrium involves no intra-neighborhood inequality, an unsegregated equilibrium has no inter-neighborhood inequality. Our earlier assumption of purely aspiration-driven altruism associated no social costs with inter-neighborhood inequality: households focused completely on the comparison with local peers. So wage inequality at the macroeconomic level did not matter for their individual, and hence aggregate, welfare. The more general formulation considered now permits both types of inequality to matter. The costs which are linked to either, together with technology-driven efficiency considerations, determine whether a social planner views segregated or unsegregated equilibria more favorably.

5.3. Utility-Enhancing Aspirations.

As indicated above, aspirations may have a positive effect on parental utility, while still generating the required complementarity in investment. We briefly consider this case. We therefore have the function $v(w, a)$, which is increasing in both arguments, concave in the first argument, and with w and a being complements.

If the complementarity is strong enough, this formulation will actually give rise to a strictly *convex* $\tilde{v}(w) \equiv v(w, w)$. For such a strong complementarity, one can again be fairly specific in comparing aspirational welfare in segregated and unsegregated equilibria. Once again, study the case of low skill bias, so that segregation is better in terms of net output. The convexity of $\tilde{v}$ implies that

$$\lambda^s v(\bar{w}^s, \bar{w}^s) + (1 - \lambda^s) v(\underline{w}^s, \underline{w}^s) > v(a^s, a^s)$$

for $a^s = \lambda^s \bar{w}^s + (1 - \lambda^s)\underline{w}^s$. Lower per-capita income under desegregation, the monotonicity of $\tilde{v}(w)$, and the concavity of v in its first argument jointly imply

$$v(a^s, a^s) > v(a^u, a^u) > \lambda^u v(\bar{w}^u, a^u) + (1 - \lambda^u) v(\underline{w}^u, a^u)$$

with $a^u = \lambda^u \bar{w}^u + (1 - \lambda^u)\underline{w}^u$. So the unsegregated equilibrium involves smaller aspirational utility than segregation, and, in view of also the latter's greater net output, a social planner would prefer segregation. This echoes the same finding for purely aspiration-driven altruism, even though the reasoning is different: here, skilled agents have a positive effect on their neighbors' utility. Because the effect is stronger, the higher is the neighbors' wage,

the overall gain is maximized by placing the skilled agents next to each other.[16] For purely aspiration-driven altruism, skilled agents actually decrease their neighbors' utility; placing them together minimizes the "damage".

Finally, the case of a high skill bias or concave $\tilde{v}(w)$ involve ambiguous comparisons. We exclude a detailed analysis.

6. Conclusion

In summary, we have investigated steady state equilibria of a model where agents' locations are given on a one-dimensional interval, and investment decisions are made by parents in the education of their children. There are two key externalities affecting investment decisions. One is an economy-wide pecuniary externality, resulting from the dependence of returns to investment on economy-wide investment ratios. Higher investment ratios in the economy lower skill premia and reduce investment incentives. The other is a local externality: earnings of residents in local neighborhoods affect investment decisions by affecting parental aspirations. These aspirations constitute one possible source of neighborhood effects; others may include a preference for conformity, or access to better schools. All of these forms of local externality induce complementarity between investment decisions at the local level, in contrast to substitutability at the economy-wide level.

We showed that steady state equilibria generally exist in which segregation arises: the interval is partitioned into subintervals in which residents all invest or do not. Unsegregated equilibria also exist in general. The macroeconomic comparison between segregated and unsegregated equilibria depends on the extent of skill-bias in the production technology. If skill-bias is low and skilled agents form a minority, segregation is associated with a higher economy-wide investment ratio and lower skill premia. The converse is true if skilled agents form a majority.

While the macroeconomic comparisons are robust with respect to the precise source of neighborhood externalities, the welfare comparisons are not. Segregation is unambiguously welfare-enhancing if skilled agents form a minority and, first, parental utility depends on children's future earnings only insofar as they deviate from aspirations, or, second, parental utility is enhanced by higher earnings of their neighbors (e.g., owing to access to better local schools). If parents also care about the earnings of their children *per se*, however, the welfare comparison is ambiguous, as segregation is associated with greater inequality *across* neighborhoods.

These results indicate that identification of the precise source of neighborhood externalities is not important if we are interested in positive analysis, e.g., the spatial structure of steady state equilibria. These are driven entirely by local complementarity properties of investment incentives. However, the source of neighborhood effects does matter when evaluating the

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[16]The same result would obtain if the local wage average reduced the costs x of investment, rather than raising its subjective benefits. Also in such a setting, an unsegregated equilibrium with $\lambda^u < 1/2$ would be dominated in terms of investment incentives by a segregated equilibrium, and hence the latter involves a higher skill ratio. In addition, the costs of those who invest are minimized by placing them together.

welfare effects of segregation. Then how the investments of neighbors affect the *level* of utilities matters, over and above their effect on marginal utilities.[17]

A number of important issues remain to be addressed. We focused entirely on steady states, and ignored issues of non-steady state dynamics. It is conceivable that the unsegregated steady states are locally unstable: shocks which lead to some local clustering of investment decisions may possibly cause the system to converge thereafter to some segregated steady state. Whereas segregation may be robust to small random perturbations. Such issues have been addressed in models of segregation based on agent mobility, following the seminal work of Schelling. It would be interesting to examine whether there is a natural tendency for non-steady-state dynamics to converge to segregated steady states in our setting, based entirely on local investment complementarities rather than agent mobility.

We also restricted attention to a one-dimensional interval for the set of all possible locations. Extension to other spatial contexts would greatly extend the applicability of the model to real-world contexts. While many of the results of our model would extend, e.g., to the circle or the plane, some would not. For instance, there cannot be a segregated equilibrium with a single cut on a circle. The example of non-existence of multi-cut segregated equilibria on the real line can be extended to the circle. Hence, it is possible that no purely segregated equilibrium exists on a circle; and the same is probably true also for the plane.

Finally, we ignored segregated equilibria which are not purely segregated, in which patterns of segregation are so fine-grained (relative to neighborhood structures) that more than two adjacent subintervals lie on any one side in the neighborhoods of some agents. These patterns can be thought of as intermediate between the unsegregated and purely segregated equilibria that we focused on in this paper. Yet other possibilities include equilibria which are unsegregated on some portions and segregated on others. The analysis of such geographic patterns is technically involved, but needs to be addressed in future work.

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[17]It is also worth mentioning that a similar analysis of the spatial structure of equilibria will obtain in contexts involving purchase of status consumption goods in a static setting, but the welfare properties of segregation will be quite distinct.

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