Collusive Market Structure Under Learning-By-Doing and Increasing Returns
Author(s): Dilip Mookherjee and Debraj Ray
Source: *The Review of Economic Studies*, Vol. 58, No. 5 (Oct., 1991), pp. 993-1009
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/2297948
Accessed: 26-12-2017 18:02 UTC

Review of Economic Studies (1991) 58, 993-1009
0034-6527/91/00570993$02.00

Collusive Market Structure Under Learning-By-Doing and Increasing Returns

DILIP MOOKHERJEE and DEBRAJ RAY
Indian Statistical Institute, New Delhi

First version received March 1988; *final version accepted November* 1990 (*Eds.*)

Learning-by-doing and increasing returns are often perceived to have similar implications for market structure and conduct. We analyse this in the context of an infinite-horizon price-setting game. Learning is shown to *not* reduce the viability of market-sharing collusion between a given number of firms, whereas intra-period increasing returns invariably does. We subsequently develop a model where the number of active firms is determined endogenously, under the assumption that the post-entry game is collusive. In this model, learning has no effect on concentration, while scale economies increase concentration.

I. INTRODUCTION

Learning-by-doing and increasing returns to scale are close cousins. The former operates through the role of *cumulative* production in lowering *future* unit costs, while the latter captures the effect of *current* production in lowering *current* unit costs.[1] It is not surprising, therefore, that these phenomena have been viewed as having similar effects on market structure; in particular, industrial concentration.

The presumption that increasing returns would tend to create concentrated industry structures is, of course, a fairly traditional one (see, for example, the studies cited in Scherer (1980)). In recent years, a number of different literatures have argued that learning-by-doing can be interpreted as a form of dynamic increasing returns that have similar effects on industry concentration and profitability. For instance, Arthur (1984, 1988) and David (1985) have argued that learning-by-doing and increasing returns cause industries to be eventually monopolized by a single technology, which would owe its dominance to a combination of "initial historical events" and the cumulative effects of increasing returns and learning-by-doing which transform a small initial lead into an unassailable advantage. Spence (1981), followed by Fudenberg and Tirole (1983), Baldwin and Krugman (1987*a*, *b*) and Dasgupta and Stiglitz (1988) have used more formal non-cooperative models of entry and product market competition to argue that learning-by-doing is similar to increasing returns in increasing entry barriers. The strategic management literature also emphasized similar ideas (e.g. see the surveys by Dutton, Thomas and Butler (1984) and Yelle (1979)). Management consultants such as the Boston Consulting Group (1970) used the "learning curve" as the basis for recommending a

1. There are other ways of describing learning-by-doing, such as the effects of cumulated *investment* on current costs (see e.g. Arrow (1962) and Sheshinski (1967)). But cumulated output appears to be the more widely used (see e.g. the discussion in Lieberman (1982)) and we follow this approach. Similarly, returns to scale can be conceptualized on dimensions other than reduction in unit costs (see Scherer (1980)), but we ignore these in this paper.

strategy of vigorous price-cutting as a competitive weapon intended to achieve market dominance.[2]

These propositions have not, however, found strong empirical support. For instance, in the context of chemical processing industries, Lieberman's (1982, 1989) analysis revealed no systematic relationship between the extent of learning and industry concentration, whereas large fixed costs tended to be associated with greater concentration. Baldwin and Krugman (1987*a*, *b*) found that the market for 16K RAM chips was significantly less concentrated than would be predicted by a theory which equated the role of learning with increasing returns. Recent management literature has also attacked the orthodox view that early output expansion would deter entry: firms using this strategy were failing to achieve the market dominance that had been hoped for (see Kiechel (1981), Day and Montgomery (1983) and Porter (1980)). Explanations of these anomalies have been sought in the presence of additional considerations such as the degree of learning spillovers (e.g. Ghemawat and Spence (1985) and Lieberman (1989)), or the nature of commitments that firms entered into (Baldwin and Krugman (1987*a*)).

Strategic models predicting a close similarity between learning-by-doing and increasing returns usually assume non-cooperative behaviour among active firms. For instance, Spence (1981) assumes a finite horizon, thereby precluding the possibility of collusion among firms entering a market. This is a common feature of most models of entry. The assumption is often justified by considerations of tractability, since equilibrium selection problems can be avoided in many finite-horizon games. Nevertheless, there is some empirical evidence suggesting the inappropriateness of this assumption. For example, in the context of markets for automobile dealership, Bresnahan and Reiss (1990) find that entering firms tend to collude with incumbents, and that the inability of incumbents to preclude such collusion perhaps explains the lack of significant entry barriers.

In this paper, we explore the implications of the alternative assumption that firms participate in an infinite-horizon price-setting game. Such contexts may enable tacit collusion among active firms. Our aim is to investigate the extent to which learning-by-doing or increasing returns facilitate collusion among a given set of firms that share a given market, and the resulting implications for industry concentration when entrants expect to collude subsequent to entry.

In Section 2 of the paper, we start by considering the question whether learning facilitates or impedes the ease with which a given number of firms can collude in setting prices. In this section, we suppose that there are no intra-period increasing returns, and concentrate exclusively on the effects of learning. We focus on forms of collusion where firms share a market equally at all dates by coordinating on a common price sequence. It turns out that in a variety of senses, the presence of learning-by-doing has no effect whatsoever on the ability of a given number of firms to sustain such forms of collusion as self-enforcing agreements (in the sense of constituting perfect equilibrium outcomes).

For instance, if firms are identical to start, with, or if demand is perfectly inelastic up to some reservation price (as in the case of auctions of procurement contracts), the nature of the learning curve does not affect their ability to sustain collusive price paths that are Pareto optimal (from the point of view of the colluding firms). In particular, Proposition 3 establishes that if the discount factor δ is less than $1-(1/n)$, where n is the number of firms, then no profitable price path can be sustained as an equilibrium outcome. On the other hand, if δ is at least $1-(1/n)$, the Pareto-optimal price path (i.e. that which maximizes a weighted sum of the present value profits of the n firms) can be

2. For a survey of this literature, as well as of related empirical work, see Mookherjee and Ray (1989).

sustained as a perfect equilibrium outcome. The maximal degree of collusion that can be supported thus has a strikingly simple form that depends solely on the discount factor and the number of firms. In addition, Proposition 4 shows that a large class of price paths can be sustained in the presence of learning if and only if they can be sustained in the corresponding no-learning world.

We subsequently explore the implications of these results for endogenous determination of the number of active firms in the industry. It is assumed that firms enter in the expectation of earning collusive post-entry profits, and that a cartel of active firms is constrained to coordinate on a price path that is not only invulnerable to the incentives of member firms, but also of passive non-member firms, to undercut the cartel price at any date. In the absence of sunk entry costs, this may be viewed as an extension of the notion of a contestable market to a collusive setting. Under conditions similar to Proposition 3, equilibrium concentration turns out to depend only on the discount factor, and in particular is independent of the nature of the learning curve. Learning, however, allows active firms to earn profits by "limit-pricing" non-active firms: the extent of industry profits thus depend on the speed of learning.

Section 3 proceeds to examine the effect of intra-period increasing returns on price collusion. We find these effects to be substantially different from those of learning-by-doing: increasing returns *always* makes collusion more difficult. The reason is the following. The presence of intra-period increasing returns expands the immediate profits of a firm that undercuts a collusive price, since such a deviation increases current output and thereby lowers current unit costs. This increases the incentive of any given firm to deviate from a collusive price, thereby restricting the range of sustainable collusive price paths. The crucial contrast with learning-by-doing is that with learning, an expansion in current output leads to a fall in *future* rather than current unit costs. By the time the cost reduction occurs, other firms have the opportunity to retaliate and punish the deviant firm suitably; the cost reduction thereby does not augment the profit from such a deviation.

Finally, Section 4 concludes with some suggestions for future research.

2. PURE LEARNING-BY-DOING

2A. *A given number of firms*

There are a given number of firms $n \geqq 2$, which produce and market a homogenous good. These firms compete in prices on a product market. Each firm is subject to a common learning curve $c(N_t^i)$. This defines the unit cost of firm i at date t as a function of its accumulated experience N_t^i which is given by $N_t^i = N_0^i + \sum_{s=0}^{t-1} Q_s^i$, where Q_s^i denotes the output of firm i at date s, and N_0^i its given level of initial experience. In this section we deal with the case of pure learning-by-doing, by abstracting from the presence of intra-period scale economies (reflected in the assumption that the current unit cost is independent of the current scale of production).

We impose the following assumptions on the nature of the learning curve:

Assumption 1. $c(N_t^i)$ is strictly decreasing and continuous, with $c(\infty) > 0$.[3]

The demand function for the product is given by $D(p)$, where p denotes the price to be paid by the buyer(s). We assume that this function satisfies the following property.

3. The assumption that c is strictly decreasing can be weakened to allow c to be weakly decreasing, without altering the main results. The case of constant returns with no learning is thus a special case of our theory. Only Proposition 5 relating to industry concentration does not apply, and the case of no learning is discussed separately there.

Assumption 2. $D(p)$ is decreasing and continuous with $D(0)<\infty$, and $D(q)=0$ for some reservation price q.

Proposition 3 will also refer to a special form of the demand function corresponding to the case of a (procurement) *auction*. In such a case, demand is constant up to a reservation price q ($D(p)=B$ for all $p \leqq q$), while it is zero above q.

We now describe the nature of price competition. At any given date, the quantity sold by firm i, denoted $Q_i(\tilde{p})$, is a function of the vector of prices $\tilde{p}=(p^1,\ldots,p^n)$ quoted by the n firms at that date. With Bertrand price competition, $Q_i(\tilde{p})=0$ if some other firm quotes a lower price than i. If i quotes the lowest price, $Q_i(\tilde{p})=D(p^i)/m$, if $m-1$ other firms also quote the lowest price p^i. This defines the single-period profit function for firm i with experience N^i: $\Pi^i(\tilde{p}, N^i)=[p^i-c(N^i)]Q_i(\tilde{p})$.

Firms participate in an infinite-horizon price-setting game, where experience levels evolve endogenously. For a given time-path of price quotations $\tilde{p}_t=(p_t^1,\ldots,p_t^n)$, firm i earns a profit of $\Pi^i(\tilde{p}_t, N_t^i)$ at date t, where $N_t^i \equiv N_0^i+\sum_{s=0}^{t-1} Q_i(\tilde{p}_s)$. All firms are assumed to use the same discount factor δ which is less than 1, so present value profits for i from date k onwards along the price path $\tilde{p}_t$ is given by $\sum_{s=k}^{\infty} \delta^{s-k}\Pi^i(\tilde{p}_s, N_s^i)$.

Throughout this paper, we restrict attention to *collusive* (*market-sharing*) *price paths* (CPP) p_t where (i) all n firms share the market by quoting the common price p_t at date t, i.e. $p_t^i=p_t$ for all i and all t, and (ii) the present value of profits (for every firm from every date onwards) is positive. Along any such CPP p_t, the profit earned by firm i at date t may be denoted $\pi(p_t, N_t^i) \equiv [p_t-c(N_t^i)][D(p_t)/n]$, where the experience level N_t^i equals $N_0^i+\sum_{s=0}^{t-1} D(p_s)/n$.

The first question to be addressed is: what is the class of CPP's that are the outcomes of some subgame-perfect equilibrium (SPE) of the infinite-horizon price-setting game played by the n firms?[4] If firm i is to increase its profit at any date t, it must undercut the collusive price p_t and capture the entire market for itself at that date. The maximal profits that it can earn at t is then given by

$$d(p_t, N_t^i)=\sup_{p<p_t}[p-c(N_t^i)]D(p).$$

For the CPP p_t to be a SPE outcome, it must be the case that

$$d(p_t, N_t^i) \leqq \sum_{s=t}^{\infty} \delta^{s-t}\pi(p_s, N_s^i) \quad \text{for all } t \text{ and } i. \tag{2.1}$$

That is, the single-period-deviation profit cannot exceed the present value profit to firm i from continuing to abide by the collusive agreement. If this condition were not satisfied then firm i could do better for itself by deviating from the collusive agreement at t, and then shutting down production from $t+1$ onwards. Clearly, (2.1) is a *necessary* condition for the CPP p_t to be a SPE outcome. The question then arises whether it is also sufficient: the following Proposition answers this in the affirmative.

Proposition 1. *A necessary and sufficient condition for a CPP p_t to be a SPE outcome is that* (2.1) *holds.*

This follows from the result that subsequent to a deviation by any firm i at any date t, there exists a continuation equilibrium set of strategies for the n firms that generates

4. For the precise definition of a subgame-perfect equilibrium, see Selten (1975) and Abreu (1988). Roughly speaking, it requires specification of strategies corresponding to the continuation subgame defined by every date and every conceivable history of past plays up to that date, and requires these to form a Nash equilibrium of the subgame.

a zero present value profit for i from $t+1$ onwards. In other words, deviating firms can be punished for a deviation in a way that they are driven down to their minimax payoff at any stage of the game. This is true irrespective of the vector of experience levels at $t+1$, or of the nature of the learning curve. Hence the left-hand side of (2.1) represents the most that firm i can obtain by deviating at t, given that this deviation is followed by imposition of the worst possible punishment on i from $t+1$ onwards. Extending the reasoning of Abreu (1988), it follows that adherence to the CPP p_t is a subgame-perfect equilibrium outcome if (2.1) is satisfied.

A detailed discussion of the argument underlying Proposition 1 is relegated to the Appendix. The result provides a convenient characterization of the class of collusive price paths that can be sustained as equilibrium outcomes. Note that the right hand side of (2.1) is monotone increasing in δ. Hence there exists a minimal discount factor (MDF) $\delta(\{p_t\})$ such that the CPP p_t is a SPE outcome (i.e. (2.1) holds) if and only if the actual discount factor is greater than or equal to this MDF. Clearly, this MDF will also generally depend on the vector of initial experience levels $\tilde{N}_0$, the learning curve $c(\cdot)$, and the market demand curve $D(\cdot)$. The effect of the learning curve on the ease with which firms can collude is then captured by the way that this critical value of the discount factor depends on the nature of the learning curve. It is this dependence that we are interested in studying.

Before proceeding further, we provide a uniform lower bound on the MDF associated with any CPP.

Proposition 2. *For any collusive price path p_t, the minimal discount factor sustaining p_t as a subgame-perfect equilibrium outcome satisfies*:

$$\delta(p_t) \geqq 1 - \frac{1}{n}.$$

Proof. Consider firm i, but drop the superscript i to simplify notation. Define $\pi^* \equiv \sup_{t \geqq 0} \pi(p_t, N_t)$. Clearly, $\pi^* > 0$. If this supremum is attained at some date T, we have, using (2.1),

$$\frac{\pi(p_T, N_T)}{1-\delta} \geqq \sum_{s=T}^{\infty} \delta^{s-T} \pi(p_s, N_s) \geqq d(p_T, N_T). \tag{2.3}$$

Now observe that by definition, $\pi(p_T, N_T) \leqq (1/n) d(p_T, N_T)$. Using this in (2.3) the result follows.

If the supremum is not attained at some finite date, there is a subsequence t_R such that $p_{t_R} \to p^*$, $N_{t_R} \to N^*$ (possibly ∞) and $\pi(p_{tR}, N_{tR}) \to \pi^*$.

Along this subsequence, (2.1) tells us that

$$d(p_{t_R}, N_{t_R}) \leqq \sum_{s=t_R}^{\infty} \delta^{s-t_R} \pi(p_s, N_s).$$

Taking lim sup on both sides and using the continuity of d,

$$d(p^*, N^*) \leqq \frac{\pi(p^*, N^*)}{1-\delta}.$$

Now use the same argument as above. ||

In order to evaluate the dependence of the MDF on the learning curve, the question that then arises is: for what specific collusive price path should this be studied? One natural candidate is the CPP that is "optimal" from the point of view of the n firms. A natural notion of optimality is the following. An *unconstrained Pareto-optimal collusive price path* p_t^* is obtained by maximizing a convex combination of the present-value profits of the n colluding firms:

$$\max_{\{p_t\}} \sum_{t=0}^{\infty} \delta^t \left[p_t - \sum_{i=1}^{n} \lambda_i c\left(N_0^i + \sum_{s=0}^{t-1} \frac{D(p_s)}{n} \right) \right] \frac{D(p_t)}{n} \tag{2.4}$$

for some set of weights $\lambda_i \geqq 0, \sum_{i=1}^{n} \lambda_i = 1$. Of course, the unconstrained Pareto-optimal price path may not be supportable as a SPE outcome: the corresponding constrained Pareto-optimal price path is obtained by solving (2.4) subject to the constraint that p_t satisfies (2.1).

The following question may now be posed: how does the MDF associated with the unconstrained Pareto-optimal price path depend on the learning curve? We start by considering environments where there is unanimity among different firms regarding which collusive path should be followed, i.e. the unconstrained Pareto-optimal CPP is independent of the weights λ_i on the profits of different firms.

Proposition 3. *Suppose either of the following conditions hold*:

(a) *All firms have identical initial experience* $N_0^i = N_0^j$, *for all* i, j.

(b) *The model is one of an auction.*

Then the minimal discount factor associated with an unconstrained Pareto-optimal collusive price path is equal to $1-(1/n)$, *i.e. is independent of the learning curve* (*as well as initial experience levels, or the market demand function*).

Proof. Given the result of Proposition 2, it suffices to show that if either (a) or (b) hold, then $\delta \geqq 1-(1/n)$ implies that any unconstrained Pareto-optimal price path can be supported as a SPE outcome.

(a) If all firms have identical initial experience N_0, and share markets at every date, then they have identical experience at every date. Hence every unconstrained Pareto-optimal price path p_t^* maximizes

$$V(N_0) \equiv \sum_{t=0}^{\infty} \delta^t \left[p_t - c\left(N^0 + \sum_{s=0}^{t-1} \frac{D(p_s)}{n} \right) \right] \frac{D(p_t)}{n}. \tag{2.5}$$

Let N_t^* denote $(N_0 + \sum_{s=0}^{t-1} D(p_s^*)/n)$, the experience of the representative firm at t; p_t^M denote the best-deviation price for a firm at t, and $\pi_M(p_t^*, c(N_t^*))$ the associated best-deviation profit.

Since the stationary policy $p_s = p_t^M$ for $s \geqq t$ is a candidate policy from t onwards in problem (2.5), it follows that (using $c(N_s^*) \leqq c(N_t^*)$ for all $s \geqq t$)

$$\begin{aligned} V(N_t^*) &\equiv \frac{1}{n} \sum_{s=t}^{\infty} \delta^{s-t} [p_s^* - c(N_s^*)] D(p_s^*) \\ &\geqq \frac{1}{n} \sum_{s=t}^{\infty} \delta^{s-t} [p_t^M - c(N_t^*)] D(p_t^M) \equiv \frac{1}{n(1-\delta)} \pi_M(p_t^*, c(N_t^*)) \\ &\geqq \pi_M(p_t^*, c(N_t^*)) \quad \text{provided } \pi_M(p_t^*, c(N_t^*)) \geqq 0, \end{aligned}$$

given that $\delta \geqq 1 - 1/n$. Hence, if the best-deviation profits are non-negative at t, no firm will wish to deviate from the unconstrained optimum at t. The same is true if the best-deviation profits are negative, because $V(N_t^*) \geqq 0$ for all t.

(b) In the case of an auction, unconstrained Pareto-optimality requires charging buyers their reservation price q in every period, since any lower price results in lower revenues without contributing to future profits by increasing the amount of learning. Hence $p_t^* = q$ for all t. Also, the maximal deviation profit for a firm i with experience N_t^i equals $n\pi(q, N_t^i)$, while the present value of continuing with the collusive path is equal to

$$\sum_{s=t}^{\infty} \delta^{s-t}\pi(q, N_s^i) \geqq \frac{\pi(q, N_t^i)}{1-\delta},$$

the inequality following from the existence of learning effects. Hence the Pareto-optimal path satisfies (2.1) if $\delta \geqq 1-(1/n)$. ||

Under the conditions of Proposition 3, the ability of the cartel to sustain collusive outcomes depends rather simply on the discount factor and the number of firms. If $\delta < 1-(1/n)$ then by Proposition 2 no collusive price path can be supported as a SPE outcome. On the other hand if $\delta > 1-(1/n)$, then the unconstrained Pareto-optimal path can be supported: in this case the unconstrained and constrained optimal paths coincide. The condition for the sustainability of optimal collusion is completely independent of the learning curve. In this sense, the presence of learning provides no impediment to price collusion at all.

To provide intuition for this result, consider first the benchmark case where there is no learning-by-doing, i.e. unit costs are independent of experience levels. Here the result can be explained as follows. Under either conditions (a) or (b), the optimal price path from the point of view of any of the colluding firms involves the stationary price which maximizes the per-period profit of a monopolist. Hence any participating firm seeking to undercut this price in any period will shade its price just enough to capture the entire market at that date. Given the absence of scale economies, the maximal deviation profit is n times its collusive market-sharing profit π^* in any given period. Since the present value of its collusive market-sharing profit is $\pi^*/(1-\delta)$, it follows that the unconstrained optimal collusive path can be sustained if and only if $\delta \geqq 1-(1/n)$.

With the introduction of learning effects, the sustainability of optimal collusion is, if anything, strengthened. We have noted above that in the absence of learning, collusive payoffs outweigh deviation profits whenever δ exceeds $1-(1/n)$. *The presence of learning further decreases the temptation to undercut the collusive price*: since future costs are lower relative to current costs, future profits from collusion are increased relative to current deviation profits. So $\delta \geqq 1-(1/n)$ continues to ensure sustainability of the maximally collusive path.[5]

The restriction of identical initial experiences, or of an auction in Proposition 3, serves to ensure that there is unanimous agreement among the firms about the optimal collusive price path. It can be shown that in the absence of such unanimity, the result may fail to be true: i.e. $\delta \geqq 1-(1/n)$ does not imply that unconstrained Pareto-optimal paths can be supported as SPE outcomes. However, this is equally true for both environments with and without learning.

To say something about the effect of learning on collusion in such contexts, we may examine how learning affects the supportability of a *given* collusive price path. For instance, is a (given) price path easier or harder to support in the presence of learning? To answer this question, we first need to settle on the appropriate benchmark cost levels

5. Note that $\delta \geqq 1-(1/n)$ continues to be necessary since the accumulation of sufficient experience leads learning effects to vanish, so that the environment "converges" eventually to a no-learning constant cost environment (where δ has to be at least $1-(1/n)$ to ensure sustainability of collusion).

for the no-learning environment to compare with a given learning environment. The reason is that the supportability of a given price sequence in a constant returns no-learning world depends on the level of unit costs. For instance, a stationary price is more difficult to support in a constant returns world, the lower the unit cost is: if $\pi(p, c)$ denotes per-period collusive profit of a firm with cost c from a price p, and $\pi_M(p, c)$ its maximal deviation profit, then the reader may verify that the ratio of collusive to deviation profit $\pi(p, c)/\pi_M(p, c)$ is non-decreasing in c. Since with the passage of time, every learning curve will eventually converge to some asymptotic level of unit cost, it follows that if we compare two learning environments with differing asymptotic cost levels, the set of supportable collusive paths will necessarily differ. Hence it is natural to compare a learning environment with a no-learning environment with identical asymptotic cost levels.

The following proposition considers a class of collusive price paths which includes all paths where prices are stationary or non-decreasing over time.

Proposition 4. *Consider any collusive price path p_t with the property that for every firm i, the associated sequence of price-cost margins $[p_t - c(N_t^i)]$ is non-decreasing over time. Then this price path is a subgame-perfect equilibrium (SPE) outcome in the presence of the learning curve $c(\cdot)$ whenever it is a SPE outcome in an environment without learning (where every firm has a constant unit cost level equal to $c(\infty)$). However, the converse is not always true.*

Proof. If p_t is supportable as a SPE outcome in the no-learning world, then for any date t and any firm i:

$$\frac{1}{n}\sum_{s=t}^{\infty} \delta^{s-t}[p_s - c_\infty]D(p_s) \geqq \pi_M(p_t, c_\infty), \tag{2.6}$$

where $\pi_M(p, c)$ denotes the maximal deviation profit of a firm with cost c from a collusive price p.

Take any firm i, and consider first the case where $\pi_M(p_t, c_t^i) \leqq 0$, where c_t^i denotes $c(N_t^i) \equiv c(N^i + \sum_{s=0}^{t-1} D(p_s)/n)$. Since by assumption price-cost margins are non-decreasing over time, and p_t is a collusive price path, the present value profit along the collusive path is positive from every date onwards. So firm i will not wish to deviate at t.

Now suppose $\pi_M(p_t, c_t^i) > 0$. At date t, let p_t^M be the best-deviation price for i, from the collusive price p_t. Then $\pi_M(p_t, c_\infty) \geqq (p_t^M - c_\infty)D(p_t^M) \geqq (p_t^M - c_t^i)D(p_t^M) = \pi_M(p_t, c_t^i) > 0$. Hence (2.6) implies

$$\frac{1}{n}\sum_{s=t}^{\infty} \delta^{s-t}\left(\frac{p_s - c_\infty}{p_t^M - c_\infty}\right)\frac{D(p_s)}{D(p_t^M)} \geqq 1.$$

Now

$$\frac{p_s - c_\infty}{p_t^M - c_\infty} = \frac{(p_s - c_s^i) + (c_s^i - c_\infty)}{(p_t^M - c_t^i) + (c_s^i - c_\infty) + (c_t^i - c_s^i)}$$

$$\leqq \frac{p_s - c_s^i}{p_t^M - c_t^i}$$

since $p_s - c_s^i \geqq p_t - c_t^i$ by assumption, $p_t \geqq p_t^M$, and the presence of learning implies

$c_t^i \geqq c_s^i \geqq c_\infty$. This implies that

$$\frac{1}{n}\sum_{s=t}^{\infty} \delta^{s-t}(p_s - c_s^i)D(p_s) \geqq (p_t^M - c_t^i)D(p_t^M) = \pi_M(p_t, c_t^i)$$

i.e. that i will not wish to deviate at t in the world with learning.

To show that the converse is not true, consider an auction where a CPP p_t is supportable in a world with learning, so for all t and i: $p_t - c_t^i \leqq 1/n \sum_{s=t}^{\infty} \delta^{s-t}(p_s - c_s^i)$. Suppose that p_t is decreasing, while $(p_t - c_t^i)$ is increasing in t. Then p_t is viable in the world with learning at any $\delta \geqq 1-(1/n)$, but with $\delta = 1-(1/n)$ it is not viable in the world without learning (since $(p_t - c_\infty)$ is decreasing). ‖

In the presence of learning, any non-decreasing price path will generate a non-decreasing sequence of price-cost margins for every firm. The above Proposition shows that (having fixed the asymptotic level of unit costs) the presence of a learning curve does not make it more difficult for such price paths to be supported as equilibrium outcomes.[6] The proof also makes it clear that the result holds even if different firms have different learning curves.

The result follows essentially from two effects: (i) as mentioned above, learning causes future costs to be lower than curent costs, thereby increasing collusive profits which accrue mainly in future periods, relative to the deviation profit which accrues only in the current period; (ii) learning causes the general *level* of costs to be lower for more experienced firms, which are more tempted to undercut a given collusive price than firms with higher costs. Hence the sustainability constraints in (2.1) that bind apply for firms with the greatest experience, for whom learning effects have been exhausted. They are identical to the supportability constraints in the benchmark no-learning world.

We finally mention another related result (established in an earlier version of this paper (Mookherjee-Ray (1987)) that demonstrates the irrelevance of the speed of learning in sustaining collusion. If a stationary collusive price can be supported as a SPE outcome with a given learning curve, it can also be supported in the presence of *any* other learning curve that converges to the same asymptotic cost. Put differently, the minimal discount factor associated with any such collusive path is independent of the nature of the learning curve or the pattern of initial experience levels. For a non-decreasing sequence, the same is true if the revenue function $R(p) = pD(p)$ is concave.

2B. *An endogenous number of firms*

We now describe implications of the foregoing results for industry concentration, under the assumption that active firms collude in setting prices, and that potential firms make entry decisions under optimistic conjectures of collusive profits subsequent to entry. Of course, forms of collusion that are feasible must respect incentive constraints elaborated below.

We restrict attention to a "free entry" environment with an infinity of potential firms that do not need to incur any sunk costs to enter; extension to the case of positive sunk costs is straightforward. Collusion betwen a cartel of active firms is constrained by two factors: (i) *Internal stability*: No member firm should have an incentive to undercut the

6. If price-cost margins are decreasing over time, then the result does not extend: for an example see Mookherjee-Ray (1987).

collusive price at any date, and (ii) *External stability*: No non-member firms should be able to earn positive profit by undercutting the cartel price at any date.[7]

Let the set of potential firms be denoted by $I=\{1,2,\ldots\}$. For any finite subset J of I, a price path p_t is said to be a *collusive price path* (*CPP*) *for J* if market-sharing by members of J (i.e. with firms in $I-J$ inactive at all dates) according to the price path p_t would yield every member of J a positive present-value profit from every date onwards. It is said to be a *subgame-perfect* (*SPE*) *outcome for J* if the path p_t can be sustained as a SPE outcome of the game played between members of J alone.

A finite subset J of I is said to be a *viable cartel* if there exists a price sequence p_t which is a CPP for J satisfying the following two conditions: (i) if J contains two or more firms, then p_t is a SPE outcome for J, and (ii) $p_t \leqq c(N_0^i)$ for all $i \notin J$ and for all t. Condition (i) represents the criterion of internal stability, and (ii) of external stability.

Finally, J is said to be an *equilibrium cartel* if J is a viable cartel, but $J \cup \{i\}$ is not a viable cartel for any $i \notin J$. That is, while J has available to it a collusive price path that is stable with respect to the incentives of both members and non-members, the same is not true when the cartel is augmented to include an additional member.

In the following Proposition, we use $[x]$ to denote the largest integer less than or equal to a real number x. Also, we use m to denote the maximal size of a cartel for which there exists a collusive price path satisfying $p_t \leqq c(N_0)$. The value of m can be calculated from the underlying demand function and the learning curve.

Proposition 5. *Suppose that all firms have identical initial experience N_0. Then there exists a unique equilibrium cartel size given by $n^* = \min\{m, [1/(1-\delta)]\}$.*

Proof. We first argue for the existence of an equilibrium cartel. Any single-firm cartel is viable by definition, since it can choose a stationary price $c(N_0)$ to make positive profit. By Proposition 2, there cannot exist a viable cartel with size exceeding $[1/(1-\delta)]$. Hence there must exist a maximal cartel size, which is thereby an equilibrium cartel size.

Suppose there is an equilibrium cartel J with size r less than n^*. Consider a cartel J^* of size $r+1$, and the following problem:

$$\max_{\{p_t\}} \sum_{t=0}^{\infty} \delta^t \left[p_t - c\left(N_0 + \sum_{s=0}^{t-1} \frac{D(p_s)}{r+1} \right) \right] \frac{D(p_t)}{r+1}$$

subject to the constraint that $p_t \leqq c(N_0)$ for all t. Since $r+1 \leqq m$ there exists a price path feasible in this problem which is a CPP for J^*. Consequently (it can be verified) that there exists a solution to the above problem, and let it be denoted by p_t^*. Using reasoning identical to that in Proposition 3, it follows that p_t^* is a SPE outcome for J^*. Hence J^* is a viable cartel, contradicting the hypothesis that J is an equilibrium cartel. ||

Equilibrium concentration is thus independent of the learning curve, except in so far as the speed of learning affects m, the maximal number of firms that can feasibly share the market without inviting entry (i.e. ignoring incentive constraints). Clearly, in any environment, the equilibrium number of firms cannot exceed this number, since active firms must make positive profit. Proposition 5 shows that the equilibrium number of firms depends only on m and the discount factor.[8]

7. Since any deviation can be followed by a punishment that awards zero profit to the deviator, the latter condition also implies the unprofitability of any attempt by a non-member to enter the market for any arbitrary number of dates as well.

8. In fact, as the speed of learning increases m may increase or decrease (the former possibility arising because faster learning implies a greater disparity between the cost of active and passive firms, enabling the former to earn higher profits in the face of the threat of entry by the latter). In some environments, thus, an increase in learning speed may actually enable *more* firms to be active.

The result relies on the existence of learning effects, as embodied in Assumption 1. These learning effects allow active firms to earn positive profits, despite the threat of entry: the profits arise from the cost advantage resulting from experience accumulated by active firms. While equilibrium concentration is independent of the learning curve, the profitability of active firms does depend on the nature of learning.

In the more usual sense of measuring the extent of entry deterrence by equilibrium concentration levels, learning thus does not serve as an entry barrier at all. This runs counter to the results of Spence (1981), or those obtained from formulating the post-entry game to be a finite-horizon non-cooperative one. Even if all firms did not have identical levels of initial experience (e.g. if one firm managed to lower its costs relative to other potential entrants by arriving earlier in the market and selling large quantities in order to climb down the learning curve), the argument of Proposition 5 can be extended under relatively weak conditions.[9]

In the absence of any learning effects, no active firm can earn any positive profit.[10] To earn profit, a firm will have to sell at a price above the common level of unit cost, in which case a non-active firm would have an incentive to enter the market and undercut the going price. Given that active firms earn zero profits and there are constant returns to scale, the equilibrium number of firms is indeterminate.

3. INCREASING RETURNS

We now modify the model of the preceding section to accommodate intra-period scale economies. Start by considering a given number ($n \geqq 2$) of firms. Assumption (1) is replaced by:

Assumption 1′. The unit cost of a firm i at date t is given by $c(N_t^i, Q_t^i)$, where Q_t^i denotes the output of i at t, and N_t^i its experience. The function c is decreasing in both arguments, continuous and bounded away from 0. Further, $c(\infty, Q)$ is a continuous and strictly decreasing function of Q.

The profit function is then modified to $\pi(p_t, N_t^i) = [p_t - c(N_t^i, D(p_t)/n)](D(p_t)/n)$, while the maximal deviation profit is now $d(p_t, N_t^i) = \sup_{p<p_t}[p - c(N_t^i, D(p))]D(p)$. With these definitions, it can be shown that Propositions 1 and 2 extend to this context (while the latter is straightforward, the former is discussed in the Appendix.)

In contrast to learning-by-doing, we shall argue that increasing returns *necessarily* makes collusion between a given number of firms more difficult to support as a self-enforcing agreement. Proposition 3 showed that with learning, the minimal discount factor required to sustain optimal collusive price paths equals $1-(1/n)$ in many circumstances, which is precisely the minimal discount factor in a comparable world without learning. We now establish that the presence of intra-period scale economies implies that the minimal discount factor associated with any collusive price path is strictly higher than this value.

Proposition 6. *Assume Assumptions* 1′ *and* 2. *The minimal discount factor required to sustain any collusive price path* p_t *is strictly greater than* $1-1/n$.

9. For instance, the result of Proposition 5 follows if (i) the definition of a viable cartel is modified to require member firms to have at least as much experience as non-members, and (ii) there exists a stationary price path enabling the m most experienced firms to earn positive (present-value) profit, where the definition of m is extended in an obvious fashion in this context.

10. This supposes, akin to the assumption of Proposition 5, that all firms have identical costs. However, if some firms have lower costs than others, they can earn positive profits. In this case the equilibrium number of firms will be determinate.

Proof. For a firm with infinite experience, let p^* be the most profitable limit point of the sequence p_t. Take any firm i, and drop superscript i hereafter to simplify notation. Let t_n be a subsequence so that $p_{t_n} \to p^*$; we know that by supportability of p_t:

$$d(p_{t_n}, N_{t_n}) \leq \textstyle\sum_{s=t_n}^{\infty} \delta^{s-t_n} \pi(p_s, N_s) \quad \text{for all } n = 1, 2, \ldots.$$

Taking limits as $n \to \infty$, we obtain

$$\begin{aligned} d(p^*, \infty) &\leqq \limsup_{n\to\infty} \textstyle\sum_{s=t_n}^{\infty} \delta^{s-t_n} \pi(p_s, N_s) \\ &\leqq \frac{\pi(p^*, \infty)}{(1-\delta)} \end{aligned}$$

implying $\delta \geqq 1 - [\pi(p^*, \infty)/d(p^*, \infty)]$. Now,

$$\begin{aligned} d(p^*, \infty) &\equiv \max_{p \leqq p^*} [p - c(\infty, D(p))]D(p) \\ &> \left[p^* - c\left(\infty, \frac{D(p^*)}{n}\right)\right] D(p^*) \\ &= n\pi(p^*, \infty) \end{aligned}$$

owing to increasing returns for infinitely-experienced firms. Hence $[\pi(p^*, \infty)/d(p^*, \infty)] < 1/n$, so $\delta > 1 - (1/n)$. ||

The intuition for this result is simple. A deviation by any firm from a collusive price enables it to capture the entire market, rather than serve a fraction of it. This expansion in scale of output leads to a fall in unit costs, in the presence of increasing returns. Thus the profits from deviating are expanded, relative to collusive profits, and firms are more tempted to undercut. An explicit lower bound on the minimum discount factor necessary to support a stationary price p is

$$\delta \geqq 1 - \frac{\pi(p, \infty)}{d(p, \infty)} \geqq 1 - \frac{1}{n} \frac{[p - c(\infty, D(p)/n)]}{[p - c(\infty, D(p))]}$$

which clearly illustrates the role of increasing returns. Note the importance of the temporal structure of increasing returns: the scale economies must be intra-period in the sense that an expansion of output in any period must lead to a fall in unit costs in *that very period.* If on the other hand, they cause unit costs to fall in future periods, we have an instance of learning-by-doing, which has already been demonstrated not to inhibit the sustainability of collusion. The reason is that a deviation by a firm in any period expands maximal deviation profits relative to collusive profits, only when the deviation lowers current-period unit costs. If the fall in unit costs is to be realized in future periods (through learning), the deviating firm can obtain no benefit, because other firms will have had the time to respond and punish the deviating firm by reverting to a relatively unprofitable equilibrium. It is the inability of other firms to respond to a deviation in the same period, that causes increasing returns to hamper the sustainability of collusion.[11]

The model of endogenous concentration described in the previous section can also be extended to the case of increasing returns. Only the condition for external stability needs to be modified as follows. Define

$$p^*(N^i) = \begin{cases} \inf\{p \mid p > c(N^i, D(p))\} & \text{if this set is nonempty,} \\ \infty & \text{otherwise.} \end{cases}$$

11. However, this result is partly due to our restriction to forms of collusion where firms share markets at every date. If they "rotate" bids and alternate market capture at successive dates, increasing returns may well make collusion easier, by exploiting the benefits of scale economies on the collusive path.

Then if i is a non-member of a cartel J choosing a collusive price path p_t, and $p_t > p^*(N_0^i)$ at any date t, then i can profitably undercut p_t. Hence external stability of the price path p_t for the cartel J requires that

$$p_t \leqq p^*(N^i) \quad \text{for all } t \text{ and for all } i \notin J.$$

Equilibrium cartels generally exist. Suppose that there are pure increasing returns, i.e. learning effects are absent (unit costs depend only on current output). If all firms have access to identical technologies, then no active firm can earn positive profit. If one firm owns a technology more efficient than the rest, this firm can earn positive profit. In either case, the presence of increasing returns implies that there cannot be more than one active firm, as any market-sharing price can be profitably undercut by a potential entrant. This is the case of a natural monopoly.[12]

Note that while increasing returns causes the industry to be concentrated, active firms may not be able to earn any profit due to the threat of entry. This is in sharp contrast with the effect of learning, which has no effect on industry concentration, but allows active firms to earn positive profits.

4. CONCLUDING COMMENTS

We suggest three directions for future research. First, our analysis has restricted attention to the class of subgame-perfect equilibria. It will be interesting to explore how the results are affected by the use of alternative equilibrium concepts. For instance, one could analyse the class of Markov equilibra where firms condition their strategies only upon the current vector of experience levels, rather than the entire history of past strategy choices. Alternatively, one could examine subgame-perfect equilibria that are renegotiation-proof in some sense.[13]

Secondly, it will be interesting to examine contexts with user externalities, e.g. where different firms offer differentiated products, and the utility of purchasing one product for any user increases with the number of other users purchasing the same product (see, for example, Katz and Shapiro (1985, 1986)). This gives firms who have succeeded in selling to more customers in the past a potential advantage over other firms. The size of past clienteles then plays the role of a state variable, rather than the volume of accumulated experience in the case of learning-by-doing.

Finally, a number of authors have stressed that the speed of learning should not be treated as exogenous, but rather something that depends on market structure in turn. For instance, firms facing greater competition on the product market may be motivated to undertake cost-reducing investments. The two-way interaction between learning and market structure deserves more attention.

APPENDIX

Here we discuss the argument underlying the proof of Proposition 1. The Proposition requires that given a CPP satisfying (2.1), any deviating firm can be punished by reverting to a subgame-perfect equilibrium in which this firm earns a profit stream with a present value equal to zero. This result is established in a previous version of this paper: for any $\delta \in (0, 1)$, and any vector of initial experience levels, there exists a subgame-perfect equilibrium giving each firm an infinite-horizon payoff of zero.

12. It is possible, however, that there are two firms with access to the "most efficient technology" that form an equilibrium cartel by sharing the market, where the technology owned by all other firms is relatively so inefficient that none of them can profitably capture the market by undercutting the cartel price.

13. For a discussion of alternative concepts of renegotiation-proof equilibria in the context of repeated games, see Bernheim and Ray (1989) and Farrell and Maskin (1989).

We do not prove this result here, for two reasons: (a) the equilibrium constructed in the proof is somewhat implausible, and (b) an alternative approach manages to avoid these difficulties and yet yield a similar result. To explain this alternative approach, we first briefly discuss some of the difficulties with the former approach.

The basic problem with the proof of the result cited in the previous paragraph is that it relies on the use of dominated strategies by high-cost firms at some stages of the game. The construction of the zero-profit equilibrium is as follows. At the initial date, all but the most experienced firm price slightly below the latter's cost, and require the most experienced firm to quote a still lower price. The latter thus makes negative profits at this date, and no deviation is posible that will earn a positive profit. From the next date onwards, the most experienced firm makes positive profits by "limit-pricing" others; these profits exactly counterbalance the losses at the initial date. All firms thus earn a zero present value profit from this outcome path; subgame perfection is ensured by re-installation of a similar outcome path following any deviation (as described by Abreu (1988) in the context of repeated games). If there is a small chance that the lowest cost firm will "tremble" and choose a sub-optimal price, high-cost firms would be unwilling to employ these pricing strategies.

One kind of equilibrium that would never be prone to this kind of difficulty is what might be called a *strict subgame-perfect equilibrium*. We define this to be a subgame-perfect equilibrium with the following additional property: for each date and each conceivable history, each firm *strictly* prefers its own best response to any possible deviation that is feasible at that node of the game. Not only do these involve players choosing undominated strategies at every stage, they are invulnerable to "trembles" or small mistakes on the part of players.

It is well-known for games with finite strategy spaces, strict equilibria satisfy all the standard refinements of Nash equilibrium. In particular, these are invulnerable to *any* conceivable pattern of "trembles" or small mistakes on the part of players. While the game discussed here is an infinite-horizon version with strategy spaces that are uncountable, a similar stability property should hold for strict equilibria in this case, with respect to a large class of sequences of "trembles" that place vanishingly small probability on actions other than the prescribed one.

We are now going to argue that collusive price paths p_t that satisfy the necessary condition for supportability with strict inequality ($\delta > \delta(p_t)$) can be supported as a strict subgame-perfect equilibrium, where $\delta(p_t)$ denotes the minimal discount factor associated with the price path p_t (i.e. the minimal value of δ for which (2.1) is satisfied). To establish this, it suffices to construct strict subgame-perfect equilibria in which all firms earn arbitrarily small (but positive) payoffs (a formal proof is provided later in this section). Following any deviation from the collusive arrangement, an equilibrium of this sort can be instituted to punish the deviating firm. The punishment equilibria are characterized by outcome paths with a two-phase structure similar to those introduced by Abreu (1986). In the first phase, the most experienced firm takes the entire market at its lowest breakeven price, while less experienced firms set a slightly higher price (thereby limiting the maximal deviation profits of the former). Thus all firms earn zero profits in this phase. In the second phase of the punishment, *all* firms participate in a market-sharing collusive arrangement and receive positive profits. The onset of the second phase is delayed enough so as to sufficiently limit the present value of the profit stream of every firm. Any deviation from this outcome path is followed by re-installation of an analogous outcome path where the deviating firm earns even smaller payoffs.

In the case of pure increasing returns, the game is a repeated one, and the second phase of the punishing outcome path can employ the original collusive path that was meant to be supported. Thus, punishments take the familiar form of a temporary price war, followed by a reversion to collusion. Deviations from prescribed strategies during the price war merely serve to extend the war.

The argument is slightly more complicated in the presence of learning, owing to the fact that the game is not a repeated one as experience levels evolve endogenously. To elaborate on this, it is convenient to make explicit the dependence of the MDF on the vector of initial experience levels. For a CPP p_t and initial experience vector $\tilde{N}$, let the minimal value of δ that enables (2.1) to be satisfied, be denoted by $\delta(p_t, \tilde{N})$.

An approach similar to that used for the pure increasing returns case can be used if the original collusive path involves a stationary price. With non-stationary paths, however, the complication is the following. Since experience levels evolve during the price war (from $\tilde{N}$ to $\hat{N}$, say), the original collusive path (which satisfied $\delta > \delta(p_t, \tilde{N})$) may no longer satisfy the necessary condition $\delta > \delta(p_t, \hat{N})$ for supportability following the price war.

To avoid this problem, we introduce the following additional assumption:

Assumption 3. The stationary price path $p_t = p_M(\infty)$ is a collusive price path for the given set of firms with initial experience vector $\tilde{N}$, where $p_M(\infty)$ denotes the monopoly price for a firm with infinite experience.

This condition requires stationary collusion at the limit monopoly price to be profitable for the n firms, and can be verified for any given parametric specification of the model. We are now in a position to prove the following result.

Proposition 1′. *Assume Assumptions* 1, 2 *and* 3. *Then any collusive price path* p_t *satisfying the condition* $\delta > \delta(p_t, \tilde{N})$ *can be supported as a strict subgame-perfect equilibrium.*

Proof. We first establish the following Lemma.

Lemma. *Assumption* 3 *and* $\delta > \delta(p_t, \tilde{N})$ *implies that the stationary collusive path* $p_t^* = p_M(\infty)$ *for all* t, *hereafter denoted* p^*, *satisfies the property that* $\delta > \delta(p^*, \hat{N})$ *for all experience vectors* $\hat{N} \geqq \tilde{N}$.

Proof of Lemma. Since p_t is a CPP with initial experience $\tilde{N}$, the argument of Proposition 2 can be used to establish that $\delta(p_t, \tilde{N}) \geqq 1-(1/n)$. Since $\delta > \delta(p_t, \tilde{N})$, it suffices to show that $\delta(p^*, \hat{N}) = 1-(1/n)$ for any $\hat{N} \geqq \tilde{N}$. Since Assumption 3 ensures that p^* is a CPP for experience vector $\tilde{N}$, it follows that it is also a CPP for experience vector $\hat{N}$. Hence by the reasoning of Proposition 2, $\delta(p^*, \hat{N}) \geqq 1-(1/n)$. We thus have to show that p^* satisfies (2.1) given $\delta = 1-(1/n)$ and the experience vector $\hat{N}$:

$$\textstyle\sum_{s=0}^{\infty} \left(1-\frac{1}{n}\right)^s \pi\left(p_M(\infty), N^i + (t+s-1)\frac{D(p_M(\infty))}{n}\right) \geqq d\left(p_M(\infty), N^i + (t-1)\frac{D(p_M(\infty))}{n}\right) \qquad (**)$$

for all i, all $N^i \geqq N_0^i$ and all t.

Taking limits as $t \to \infty$, (**) requires that

$$\textstyle\sum_{s=0}^{\infty} \left(1-\frac{1}{n}\right)^s \pi(p_M(\infty), \infty) \geqq d(p_M(\infty), \infty)$$

$$= \sup_{p<p_M(\infty)} [p - c(\infty)] D(p)$$

$$= n\pi(p_M(\infty), \infty),$$

which is satisfied since $\sum_{s=0}^{\infty} (1-1/n)^s = n$.

Further, for all $s \geqq 0$:

$$\frac{\pi\left(p_M(\infty), \hat{N}^i + (t+s-1)\frac{D(p_M(\infty))}{n}\right)}{d\left(p_M(\infty), \hat{N}^i + (t-1)\frac{D(p_M(\infty))}{n}\right)} \geqq \frac{\pi(p_M(\infty), \infty)}{d(p_M(\infty), \infty)},$$

since the left-hand side of this expression is at least equal to

$$\frac{\pi\left(p_M(\infty), \hat{N}^i + (t-1)\frac{D(p_M(\infty))}{n}\right)}{d\left(p_M(\infty), \hat{N}^i + (t-1)\frac{D(p_M(\infty))}{n}\right)},$$

and since $[(p-c)[D(p)/n]/\sup_{p'<p} (p'-c)D(p')]$ is a nondecreasing function of c. Hence (**) is satisfied, thus concluding the proof of the lemma. ‖

Now return to the proof of the main Proposition. For simplicity of notation, consider the case of 2 firms. Let $N^F \equiv \min(N_0^1, N_0^2)$ and $N^L \equiv \max(N_0^1, N_0^2)$. Call L the *leader* and F the *follower.* We start by constructing a family of outcome paths as follows. Consider any $\hat{N} \geqq \tilde{N}$, and any $\varepsilon > 0$. We shall show that there exists an outcome path $\Theta(\varepsilon, \hat{N})$ such that

(i) both agents obtain a payoff on $\Theta(\varepsilon, \hat{N})$ that is strictly less than ε.
(ii) by deviating from $\Theta(\varepsilon, \hat{N})$ at any date t, each agent obtains strictly less at t, compared to his infinite horizon payoff from t onwards.

Observe that by condition (ii), each agent's payoff on $\Theta(\varepsilon, \hat{N})$ must be positive.

To construct such a path, define for the given $\hat{N} \geqq \tilde{N}$,

$$a \equiv V(p^*, N^L)$$

$$b \equiv V(p^*, \infty),$$

where $V(p^*, N)$ denotes the present value profit of a firm with initial experience N along the collusive path p^*. By Assumption 3, p^* is a collusive price path from $\tilde{N}$. Hence $a>0$ and $b>0$. Next given $\varepsilon>0$, choose T so that $\delta^T b<\varepsilon$. Given this choice of T, choose $\mu(N)$ for any given N so that

$$(p-c(N))D(p)<\delta^T a \quad \text{for all } p\in[c(N), c(N)+\mu(N)].$$

We are now in a position to describe $\Theta(\varepsilon, N)$. For $t=0,\ldots, T-1$, the leader charges $c(N_t)$, where $N_0\equiv N^L$ and $N_{t+1}\equiv N_t+D(c(N_t))$, $t=0,\ldots, T-1$. The follower charges $c(N_t)+\mu(N_t)$, $t=0,\ldots, T-1$. For all $t\geqq T$, both leader and follower charge the price $p_M(\infty)$.

The reader can now check that under the assumption of this proposition, $\Theta(\varepsilon, \hat{N})$ satisfies conditions (i) and (ii).

We now claim that starting from the experience vector $\hat{N}$, any such outcome path is supportable as a strict subgame-perfect equilibrium. For suppose that a deviation from $\Theta(\varepsilon, N)$ occurs at time t. Let $\hat{N}'$ be the experience vector following the deviation. Define ε' to be the difference between the infinite-horizon payoff at time t to the deviating agent, if he had persisted with $\Theta(\varepsilon, \hat{N})$, and the value of the current return due to the deviation. By condition (ii), $\varepsilon'>0$. At time $t+1$, start up the outcome path $\Theta(\varepsilon', \hat{N}')$. Use a similar method of switching to a new path should any deviation occur from $\Theta(\varepsilon', \hat{N}')$. The reader can easily verify that the strategy pair implied by these rules forms a strict subgame-perfect equilibrium.

To complete the proof, consider any collusive price path p_t with $\delta>\delta(p_t, \tilde{N})$. Define $\hat{\varepsilon}$ by

$$\hat{\varepsilon}\equiv \inf_{i=1,2;\, t\geqq 0}\{V(p_t, N_t^i)-d(p_t, N_t^i)\}. \tag{*}$$

Because $\delta>\delta(p_t, \tilde{N})$, we have $\hat{\varepsilon}>0$. Now suppose that any deviation takes place, with a post-deviation experience vector of $\tilde{N}'$. Follow up this deviation with the outcome path $\Theta(\hat{\varepsilon}, \tilde{N}')$. Now use (*) and the fact that $\Theta(\hat{\varepsilon}, \tilde{N}')$ is supportable to conclude that p_t is supportable as a strict subgame-perfect equilibrium. ||

Acknowledgement. This is a considerably revised version of an earlier paper entitled "Dynamic Price Games with Learning-by-Doing". We are grateful to Dilip Abreu, Sushil Bikhchandani, James Mirrlees, John Moore, John Roberts, Robert Wilson and the referees for helpful comments.

REFERENCES

ABREU, D. (1988), "On the Theory of Infinitely Repeated Games With Discounting", *Econometrica*, **56**, 383-396.

ABREU, D. (1986), "Extremal Equilibria of Oligopolistic Supergames", *Journal of Economic Theory*, **39**, 191-225.

ARROW, K. (1962), "The Economic Implications of Learning-by-Doing", *Review of Economic Studies*, **29**, 155-173.

ARTHUR, B. (1984), "Competing Technologies and Lock-In by Historical Small Events: The Dynamics of Allocation under Increasing Returns" (CEPR Publication, Stanford University).

ARTHUR, B. (1988), "Self-Reinforcing Mechanisms in Economics," in Anderson, P. W. and Arrow, K. J. (Eds), *The Economy as an Evolving Complex System.* (New York: Addison-Wesley).

BALDWIN, R. and KRUGMAN, P. (1987*a*), "Market Access and International Competition: A Simulation Study of 16 K Random Access Memories," in Feenstra, R. (Ed.) *Empirical Studies in International Trade* (Cambridge, Mass.: MIT Press).

BALDWIN, R. and KRUGMAN, P. (1987*b*), "Modelling International Competition in High Technology Industries: Lessons from Aircraft and Semiconductors" (mimeo, MIT).

BERNHEIM, B. D. and RAY, D. (1989), "Collective Dynamic Consistency in Repeated Games", *Games and Economic Behavior* **1**, 295-326.

BRESNAHAN, T. and REISS, P. (1990), "Entry in Monopoly Markets", *Review of Economic Studies*, **57**, 531-553.

BOSTON CONSULTING GROUP (1970) *Perspectives on Experience* (Boston, Mass.: Boston Consulting Group).

DASGUPTA, P. and STIGLITZ, J. (1988), "Learning By Doing, Market Structure and Industrial and Trade Policies", *Oxford Economic Papers*, **40**, 246-268.

DAVID, P. (1985), "Clio and the Economics of QWERTY", *American Economic Review, Papers and Proceedings*, **75**, 332-337.

DAY, G. and MONTGOMERY, D. (1983), "Diagnosing the Experience Curve", *Journal of Marketing*, **47**, 44-58.

DUTTON, J., THOMAS, A. and BUTLER, J. (1984), "The History of Progress Functions as a Managerial Technology", *Business History Review*, **58**, 204-237.

FARRELL, J. and MASKIN, E. (1989), "Renegotiation in Repeated Games", *Games and Economic Behavior*, **1**, 327-360.

GHEMAWAT, P. and SPENCE, M. (1985), "Learning Curve Spillovers and Market Performance", *Quarterly Journal of Economics*, **100**, 839-852.

KATZ, M. and SHAPIRO, C. (1985), "Network Externalities, Competition and Compatibility", *American Economic Review*, **75**, 424-440.

KATZ, M. and SHAPIRO, C. (1986), "Technology Adoption in the Presence of Network Externalities", *Journal of Political Economy*, **94**, 822-841.

KIECHEL, W. (1981), "The Decline of the Experience Curve", *Fortune*, October 3, 139-145.

KOHLBERG, E. and MERTENS, J. F. (1986), "On the Strategic Stability of Equilibria", *Econometrica*, **54**, 1003-1038.

LIEBERMAN, M. (1982) *The Learning Curve, Pricing and Market Structure in the Chemical Processing Industries* (Harvard University, Ph.D thesis).

LIEBERMAN, M. (1989), "The Learning Curve, Technology Barriers to Entry, and Competitive Survival in the Chemical Processing Industries", *Strategic Management Journal* (forthcoming).

MOOKHERJEE, D. and RAY, D. (1987), "Collusive Market Structure with Learning-By-Doing and Increasing Returns" (Discussion Paper No. 884R, Graduate School of Business, Stanford University).

MOOKHERJEE, D. and RAY, D. (1989), "Learning-By-Doing and Industrial Market Structure: An Overview", in Dutta, B. et al (Eds), *Theoretical Themes in Development Economics* (New Delhi: Oxford University Press) (forthcoming).

PORTER, M. (1980) *Competitive Strategy* (New York: Free Press).

SCHERER, F. (1980) *Industrial Market Structure and Economic Performance* (*2nd Edition*) (New York: Rand Mc-Nally).

SELTEN, R. (1975), "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games", *International Journal of Game Theory*, **4**, 25-55.

SHESHINSKI, E. (1967), "Tests of the Learning by Doing Hypothesis", *Review of Economics and Statistics*, **49**, 568-578.

SPENCE, M. (1981), "The Learning Curve and Competition", *Bell Journal of Economics*, **12**, 49-70.

YELLE, L. (1979), "The Learning Curve: Historical Review and Comprehensive Survey", *Decision Sciences*, **10**, 302-327.